

## Lecture 9: 'Leftovers' from No Free Lunch

# No Free Lunch Theorem

- ▶ We have now seen theoretical arguments for why no optimisation algorithm can be claimed to be superior over all possible problems
  - ▶ Hence, no algorithm can be expected to perform better than random, or enumerative, search, given no information about the optimisation problem at hand
- ▶ This result is the equivalent, for search and optimisation, of the *Law of Conservation of Generalisation* for classification algorithms
- ▶ Is the No Free Lunch theorem the definitive statement on optimisation?
  - ▶ No, it relies on some stringent assumptions

# No Free Lunch Assumptions

- ▶ All problems in the set of possible problems are assumed to be equally likely
  - ▶ When we calculate overall performance of algorithm  $A$  we calculate an unweighted sum or mean, e.g.

$$\bar{A} = \frac{\sum_{f \in \mathcal{R}^S} A_f}{|\mathcal{R}^S|}$$

- ▶ If we face different problems with different probabilities things are different
  - ▶ Algorithms *will* have different overall performance according to some measure
  - ▶ In the real world different problems will be more likely than others

# No Free Lunch Assumptions

- ▶ The set of possible problems is assumed to be *closed-under-permutation* (c.u.p)
  - ▶ If we permute one of the problems we are guaranteed to get a problem still in the same set
- ▶ We can calculate the proportion of random subsets of  $\mathcal{R}^{\mathcal{S}}$  that are c.u.p.

$$\frac{2^{\binom{|\mathcal{R}|+|\mathcal{S}|-1}{|\mathcal{R}|}} - 1}{2^{(|\mathcal{S}||\mathcal{R}|)} - 1}$$

- ▶ For very small  $\mathcal{R}$  and  $\mathcal{S}$  this is already negligible
- ▶ We can also show that most interesting problem classes are almost guaranteed not to be c.u.p.

## Further Reading

- ▶ Igel and Toussaint (2003) Recent results on No-Free-Lunch theorems for optimization. arXiv:cs.NE/0303032 v1.
- ▶ Igel and Toussaint (2003) On classes of functions for which No Free Lunch results hold. Information Processing Letters 86, 317-321.