COMSM0302 - WEEK 8 SOLUTIONS

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1. Exercise 1 - Schemata

Schema	**Instances**	**Order**	**Length**
S_1	2	1	0
S_2	2	1	0
S_3	3	2	1
S_4	2	3	3
S_5	2	2	6
S_6	1	4	3

2. Exercise 2 - Limiting Distribution of a 2-State Markov Process

$$q = \begin{pmatrix} \frac{5}{12} \\ \frac{7}{12} \end{pmatrix}$$

3. Exercise 3 - Limiting Distribution of a 3-State Markov Process

$$q = \begin{pmatrix} \frac{1}{2} \\ \frac{3}{8} \\ \frac{1}{8} \end{pmatrix}$$

4. Exercise 4 - Absorbing States in a Markov Processes

$$Q^\infty = \begin{pmatrix} 1 & 0 & 0.4706 & 0.4118 \\ 0 & 1 & 0.5294 & 0.5882 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Expected time to absorption from states $a = \begin{pmatrix} 1.8235 & 1.4706 \end{pmatrix}$.

5. Exercise 5 - Mixing Matrix for 1X

$$M(0) = \begin{pmatrix} 1 & 0 & \frac{1}{2} & 0 & 1 & 0 & \frac{1}{2} & 0 \\ 1 & 0 & \frac{1}{2} & 0 & 1 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_{6,1}(7) = \frac{1}{2}$$

6. Exercise 7 - Convergence of Random Search

Let $X_0, X_1, X_2, \ldots$ be the sequence of values returned by the objective function $f(x)$ during the random search. Then let the sequence $D_0, D_1, D_2, \ldots$ be generated by the $D_t = D(X_t)$ which returns the maximum value of $f(x)$ minus the value of the variable a in the random search algorithm. The function D must be bounded, as the function $f(x)$ is bounded by the highest value solution in the search space (N.B. if the search space if infinite we can still assume $f(x)$ is bounded if it is implemented on a computer, as the data-type returned by $f(x)$ will typically be finite). Because the value of a can never decrease in the random search algorithm, we have $E(D_{t+1}|X_t) \leq D_t$ and the sequence is a *non-negative supermartingale* which converges almost surely to 0 (i.e. the value of the best solution found by the algorithm so far converges almost surely to the global optimum).