COMSM0302 - WEEK 8 EXERCISES

JAMES MARSHALL
JANUARY 18, 2007

1. Exercise 1 - Schemata

Consider the following population of 4 chromosomes, C_1 to C_6, and the 6 schemata

C_1	11101111
C_2	10101011
C_3	00010100
C_4	01000011

definitions, S_1 to S_6.

S_1	1*******
S_2	0*******
S_3	******11
S_4	***0*01*
S_5	1*****1*
S_6	1110****

What is the order and length of each schema? How many instances of each schema are there in the population?

2. Exercise 2 - Limiting Distribution of a 2-State Markov Process

Given the following transition matrix, calculate the limiting probability distribution over the different states, i.e. find the vector q such that $Qq = q$.

$$Q = \begin{pmatrix} \frac{1}{5} & \frac{4}{7} \\ \frac{4}{5} & \frac{3}{7} \end{pmatrix}$$

Hint: As the transition matrix is only order 2×2, the matrix q giving the limiting state distribution can be calculated by solving the matrix equation $Q \begin{pmatrix} x \\ 1-x \end{pmatrix} = \begin{pmatrix} x \\ 1-x \end{pmatrix}$

3. Exercise 3 - Limiting Distribution of a 3-State Markov Process

Given the following transition matrix, calculate the limiting probability distribution over the different states, i.e. find the vector q such that $Qq = q$.

$$Q = \begin{pmatrix} \frac{1}{4} & 1 & 0 \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{4} & 0 & 0 \end{pmatrix}$$

Hint: You can use Matlab to calculate the eigenvalues and eigenvectors of a matrix, with the command `eig`. N.B. you must normalise the eigenvector with eigenvalue 1 to obtain a unit vector (i.e. a probability distribution)

4. Exercise 4 - Absorbing States in a Markov Processes

Given the following transition matrix, calculate the infinite-time limit of the transition matrix, and the vector of times to absorption from each possible state.

$$Q = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{7} & 0 \\ \frac{1}{4} & 1 & \frac{2}{7} & 0 \\ \frac{1}{4} & 0 & \frac{1}{7} & 0 \\ \frac{1}{4} & 0 & \frac{3}{7} & 1 \end{pmatrix}$$

Hint: You can use Matlab to calculate matrix inverses, with the command `inv`

5. Exercise 5 - Mixing Matrix for 1X

Calculate the mixing matrix $M(0)$ for one-point crossover operating on binary chromosomes of length $\ell = 3$, with the row chromosome being the first parent. Use this matrix to determine $M_{6,1}(7)$.

6. Exercise 6 - Non-Convergence of GA with Mutation

Run multiple replicates of a simple GA with fitness-proportional selection and mutation at rate $\mu = 0.1$, but no crossover, on binary strings of length $\ell = 2$. Population size should be $N = 4$, and the fitness function is $f(00) = 1$, $f(01) = 2$, $f(10) = 3$ and $f(11) = 4$. Observe the proportion of replicates in which, at each GA iteration, the population contains only copies of the optimal solution and the proportion in which the population contains no copies of the optimal solution.

7. Exercise 7 - Convergence of Random Search

Show that for the following random algorithm searching in the space $\mathcal{C}$ the value of a converges almost certainly to the maximum of the positive function f.

Data: a — value of best point found so far
$a \leftarrow 0$;
repeat
  Generate random point $x \in \mathcal{C}$;
  if $f(x) > a$ **then**
    $a \leftarrow f(x)$;
  end
until *forever*;