

# Implementing LEGO

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Roberto Trifiletti



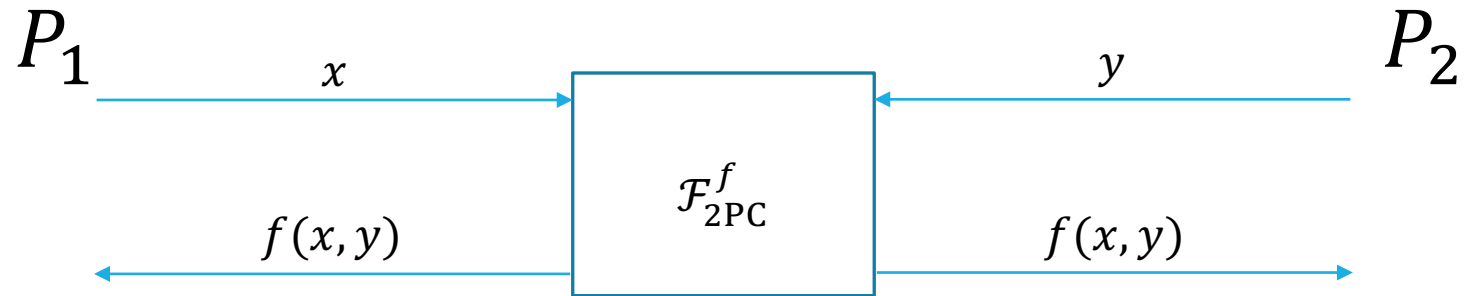
# Outline

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1. Secure Two-party Computation Based on Garbled Circuits.
2. Implementing TinyLEGO using Efficient XOR-Homomorphic Commitments [NST17].
3. DUPLO, a Unified Approach to Cut-and-Choose [Ongoing work].

# Secure 2PC

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Nothing but the output  $f(x, y)$  is revealed to the parties.

Task: realize above scenario using a cryptographic protocol.

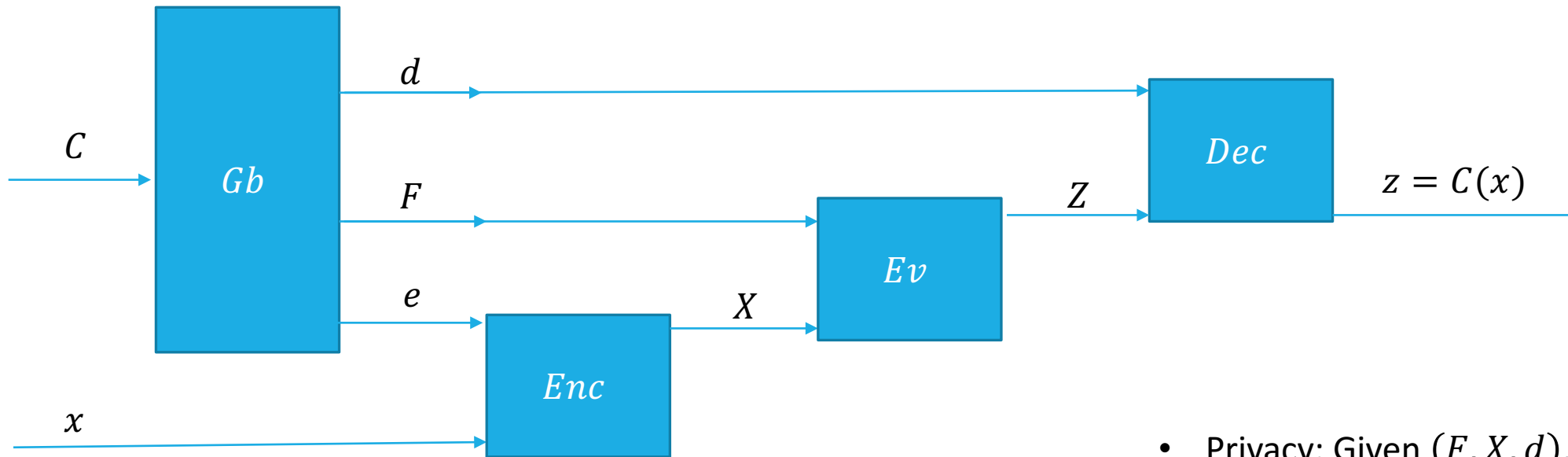
Powerful: can build most other crypto from secure computation.

Applications:

- Privacy preserving data analysis
- Company benchmarking
- Satellite collision detection

# Garbling Schemes [BHR12]

$$G = (Gb, Enc, Ev, Dec)$$



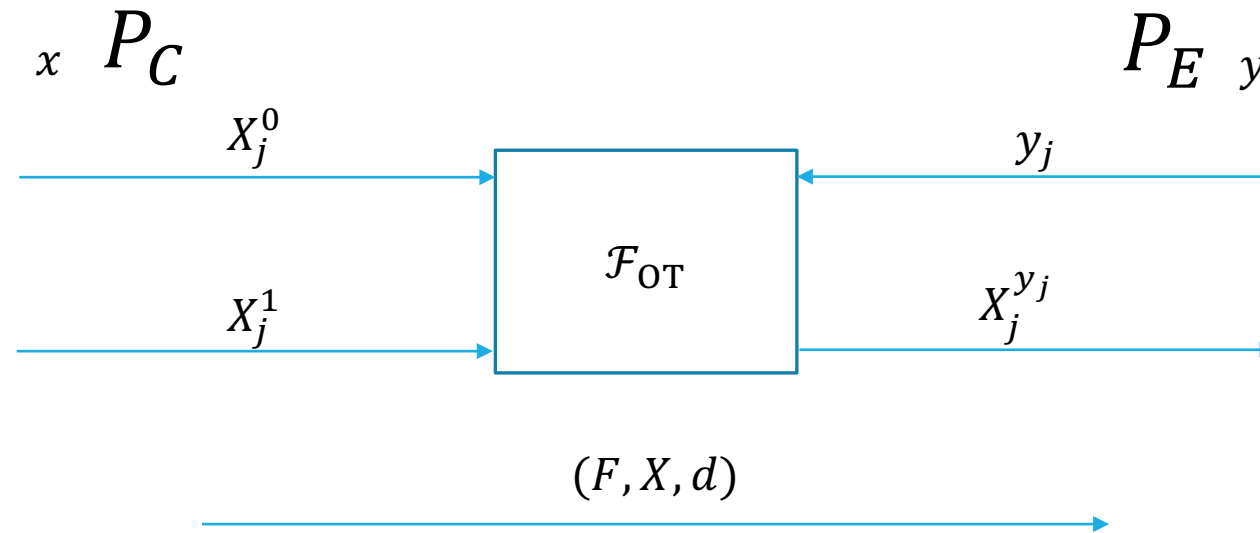
- Privacy: Given  $(F, X, d)$ , only learn  $C(x)$ .
- Optimization: Free-XOR [KS08], no data transfer for XOR gates.
- Using Half-Gates [ZRE15],  $2k$  bits per AND gate.

# Semi-honest: Yao's garbled circuits

$$(F, e, d) \leftarrow Gb(C)$$

$$(X_1^0, X_1^1, \dots, X_n^0, X_n^1) \leftarrow e$$

$$X \leftarrow Enc(x, e)$$



$$Y \leftarrow (X_1, X_2, X_n)$$

$$Z \leftarrow Ev(F, Y)$$

$$z \leftarrow Dec(Z, d)$$

$$z = C(x, y)$$

# Malicious adversary

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Yao's garbled circuits completely break against *malicious* behavior

- $P_C$  can garble  $C' \neq C$  and  $P_E$  would never know.
- Selective Failure Attack: Make  $P_E$  abort *depending* on his input (thus leaking information about  $y$ ).

# Malicious: “Standard” Cut-and-choose

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## Main idea

- Send multiple garblings  $F_1, F_2, \dots, F_m$ , check some, evaluate the rest.
- Not trivial, but possible with care.

## Replication cost

- [Bra13, HKE13, Lin13]:  $s$  circuits gives  $2^{-s}$  security.
- 40-80x blowup in communication/computation.

## Amortization

- [HKKKM14, LR14, LR15, RR16]:  $O(s/\log(\#C))$  circuits gives  $2^{-s}$ , i.e. cut-and-choose overhead is amortized over multiple individual computations of  $C$ .
- [NO16] Cross&Clean  $O(1)$  circuits, mostly theoretical result.

# LEGO

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[NO09] introduced LEGO technique for maliciously secure 2PC based on cut-and-choose of Garbled Circuits.

Considers gates instead of circuits for cut-and-choose

- Asymptotic improvement,  $\mathcal{O}(s/\log(|C|))$  vs  $\mathcal{O}(s)$ .
- Allows preprocessing that is independent of  $C$ .
- Requires “soldering” individual gates to form a circuit using homomorphic commitments.

[NO09] downsides

- Expensive public-key operations for each gate of the circuit.
- Incompatible with optimizations of Yao’s garbled circuits.

[FJNNO13, FJNT15, FJNT16] Improvements

- Eliminate public-key operations for each gate.
- Compatible with all known optimizations.
- Efficient XOR-homomorphic commitment scheme based on ECC and OT.
- More efficient bucketing strategy.



# How to Solder

Free-XOR:  $L_j^1 = L_j^0 \oplus \Delta$ ,  $R_j^1 = R_j^0 \oplus \Delta$ ,  $O_j^1 = O_j^0 \oplus \Delta$  for all  $\text{AND}_j$  gates.



- A soldering between two wires is then the difference between the 0-labels (truth value):

$$S_{ij}^L = L_i^0 \oplus L_j^0, \quad S_{ij}^R = R_i^0 \oplus R_j^0, \quad S_{ij}^O = O_i^0 \oplus O_j^0$$

- Ex: When learning labels  $X_0^b$  can now compute

$$X_0^b \oplus S_{01} = (X_0^0 \oplus b \cdot \Delta) \oplus (X_0^0 \oplus X_1^0) = X_1^0 \oplus b \cdot \Delta = X_1^b.$$

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2. Implementing TinyLEGO using Efficient XOR-Homomorphic Commitments [NST17].
3. DUPLO, a Unified Approach to Cut-and-Choose [Ongoing work].

# [NST17]

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Efficient C++ implementations of [FJNT16] commitment scheme and [FJNT15] LEGO protocol, both heavily utilizing parallelization for maximal performance.

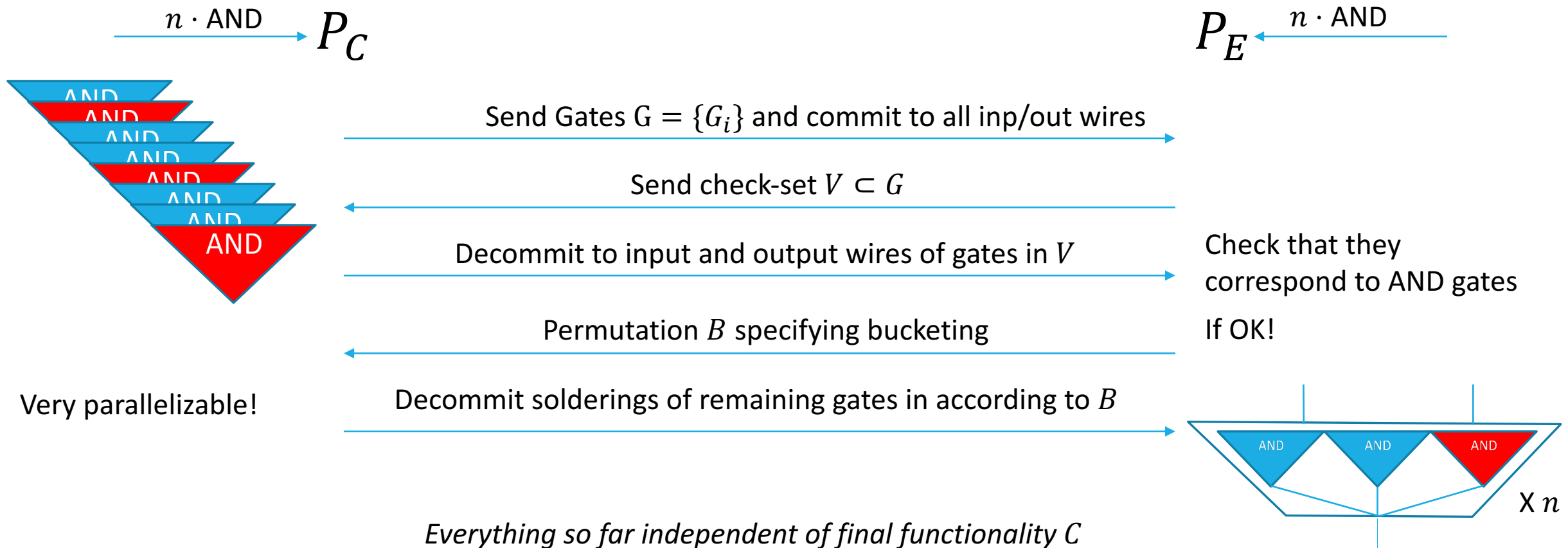
New approach to dealing with Selective Failure Attack on  $P_E$ 's input

- No increase of  $P_E$ 's input as with XOR-tree ( $s$ -probe matrix) approach.
- Saves on #OTs, communication and computation.
- XOR-homomorphic commitments combined with *globally correlated*  $\Delta$ -ROT.

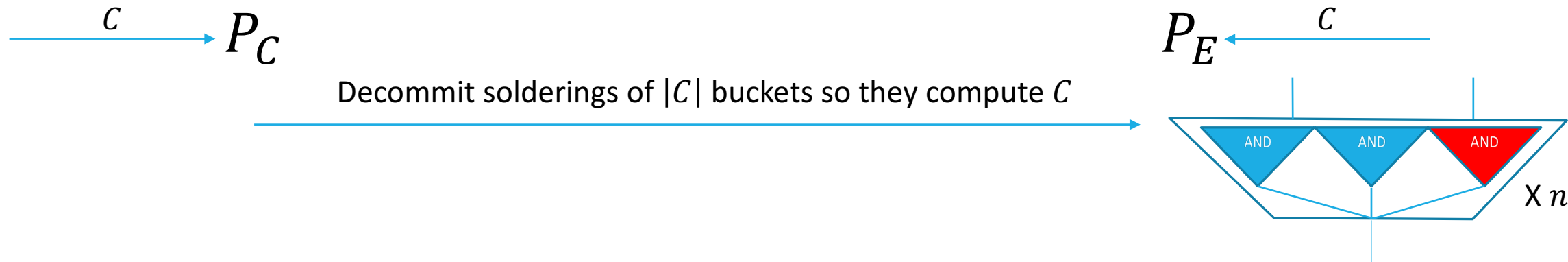
Tightened the analysis of current construction of  $\Delta$ -ROT (all [IKNP03]-like OT extensions)

- $\sim 5x$  reduction in  $\Delta$ -ROT communication cost for  $k = 128, s = 40$ .
- Core part of [NNOB12] “TinyOT”. Benefits recent works of [WRK17a, WRK17b, HSV17].

# Phase 1: Preprocessing



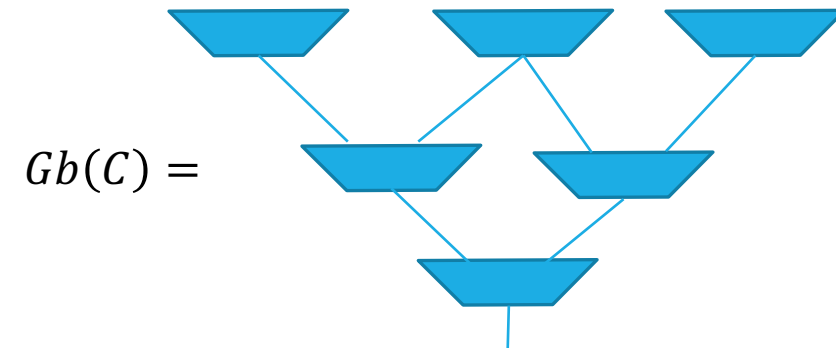
# Phase 2: Function soldering



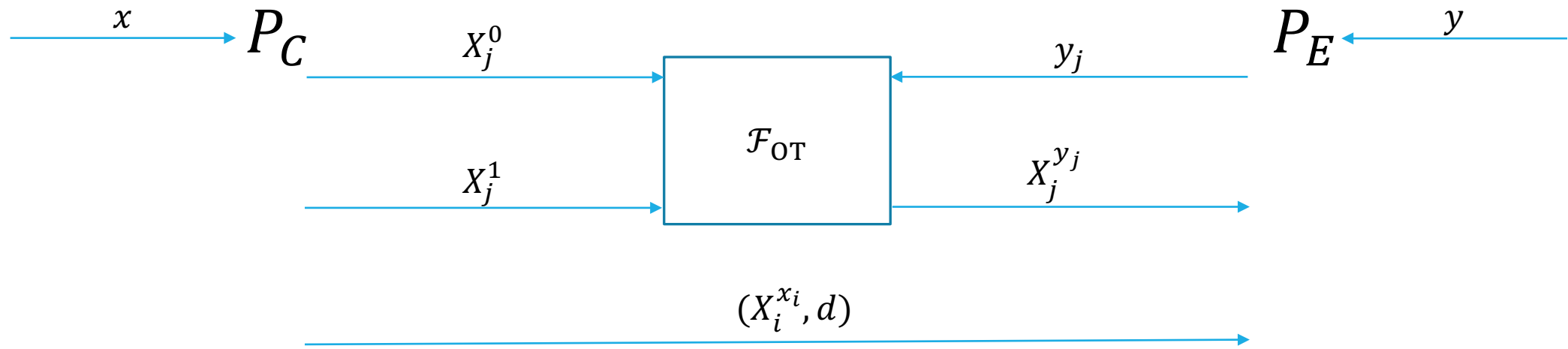
Data transfer cost:

- $2 \cdot |C|$  decommits.
- With [FJNT16] commit scheme:  $2 \cdot |C| \cdot k + c$  bits ( $\sim 1$  garbled circuit).
- Non-LEGO:  $\mathcal{O}(s \cdot |C| \cdot k)$ .

$k$  comp. security param,  $s$  stat. security param.

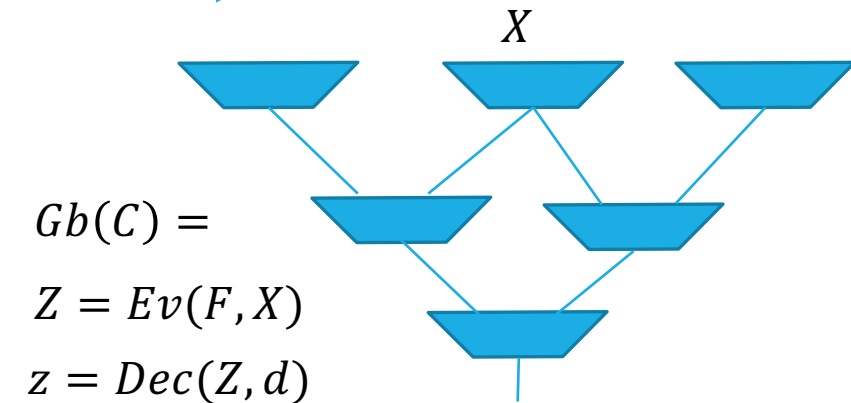


# Phase 3: Evaluation



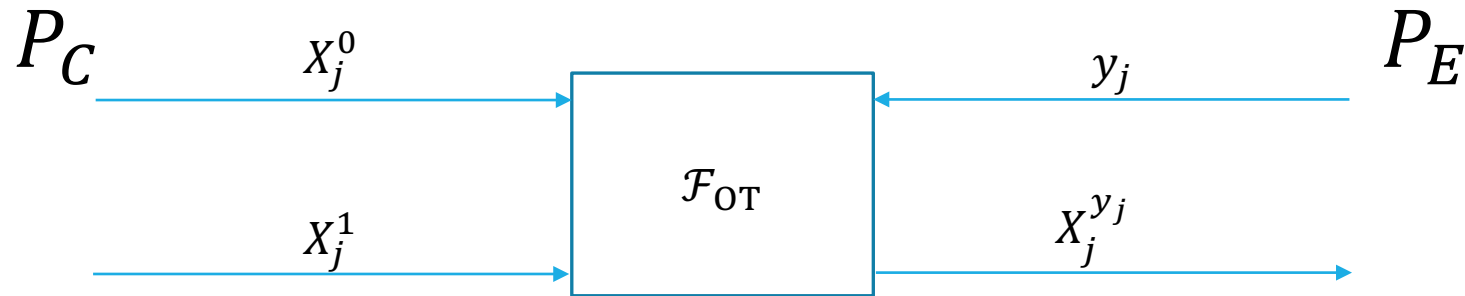
## Highlights:

- LEGO: Single set of input labels vs. non-LEGO: one per eval circuit.
- Optimal 2 rounds (3 if  $P_C$  gets output).
- Computation: Evaluating  $\mathcal{O}(s/\log(|C|))$  garbled circuits.



# Selective Failure Attack [MF06, KS06]

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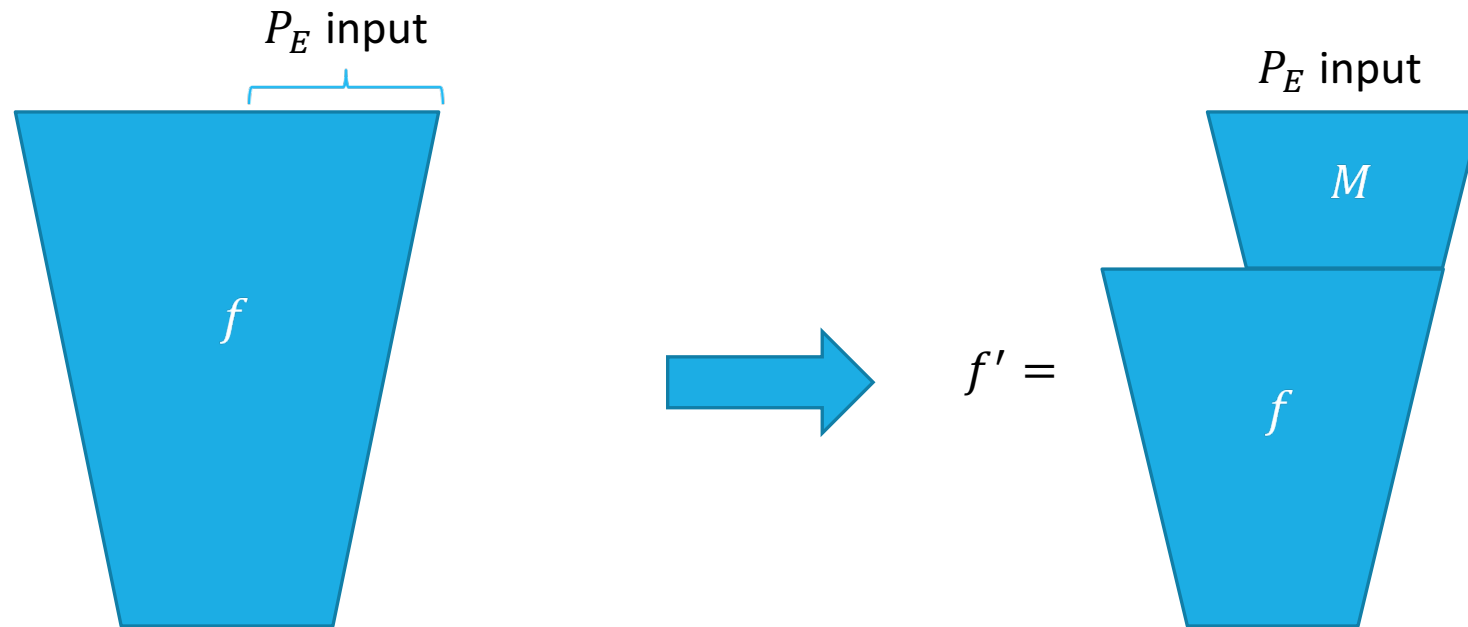


However, say  $P_C$  inputs a value  $\widehat{X}_j^0 \notin \{X_j^0, X_j^1\}$  for one of the input OTs.

- If  $P_E$  does not abort => input is 1.
- If  $P_E$  aborts => input is 0.
- In general,  $P_C$  can learn  $l$  input bits with probability  $2^{-l}$ .
- Hard to deal with due to nature of OT ( $P_E$  learns only one value).

# Standard approach: XOR-tree ( $s$ -probe matrix) [LP07, sS11]

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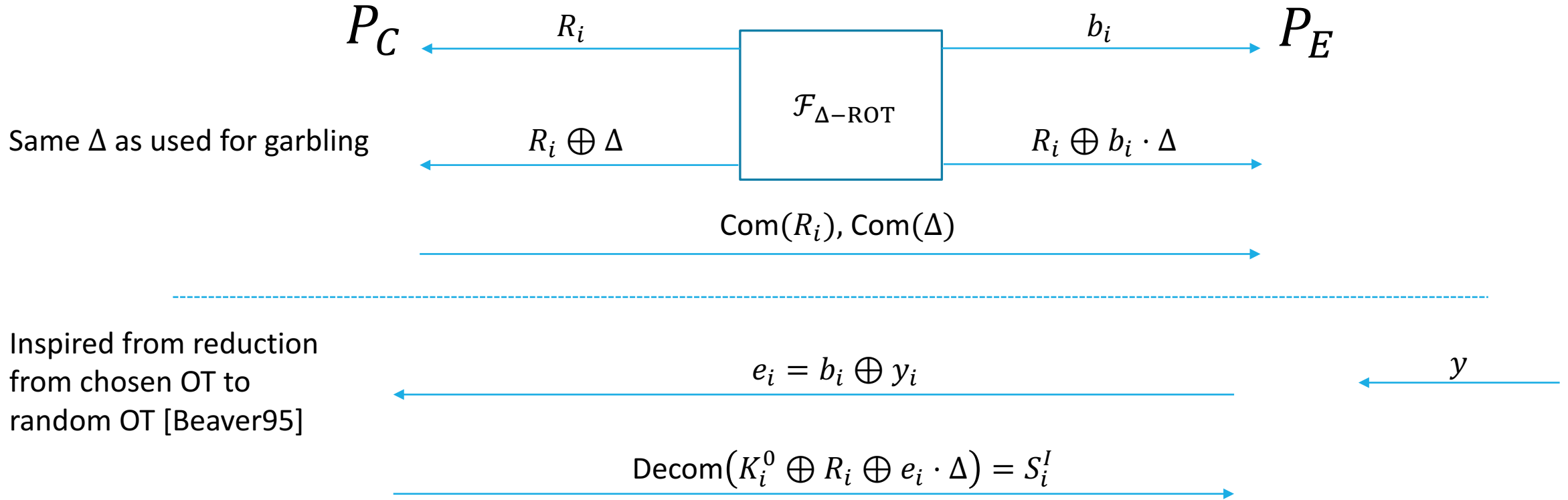


Guarantee: Learning up to  $s$  input bits of  $P_E$  for new function  $f' = M \circ f$  does not leak anything about the real input  $y$  to  $f$ .

- Increases #OTs, communication and computation time.

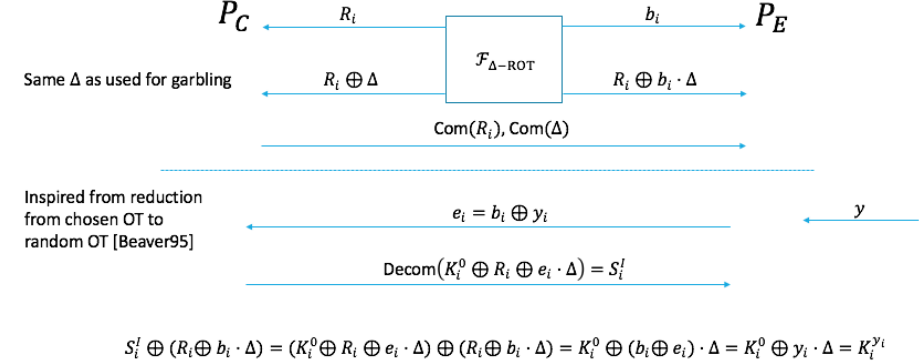


# New selective failure approach [NST17]



$$S_i^I \oplus (R_i \oplus b_i \cdot \Delta) = (K_i^0 \oplus R_i \oplus e_i \cdot \Delta) \oplus (R_i \oplus b_i \cdot \Delta) = K_i^0 \oplus (b_i \oplus e_i) \cdot \Delta = K_i^0 \oplus y_i \cdot \Delta = K_i^{y_i}$$

# Why secure?

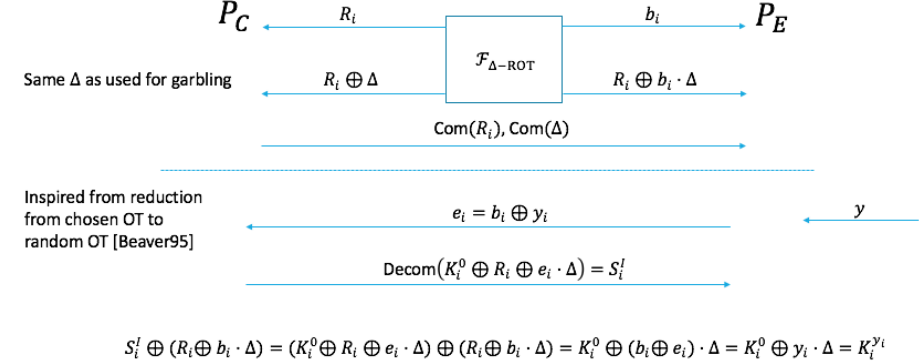


Intuitively,  $\text{Decom}(K_i^0 \oplus R_i \oplus e_i \cdot \Delta)$  is either correct for both, or neither input labels.

Wrong  $\Delta$  commit:

- Com( $\hat{\Delta}$ ) where  $\hat{\Delta} \neq \Delta$ , then  $S_i^I \oplus (R_i \oplus b_i \cdot \Delta) \notin \{K_i^0, K_i^1\}$  if  $b_i = 1$ .
- Solution: Cut-and-choose on  $s$  OTs. If cheating gets caught with. prob  $1 - 2^{-s}$ .

# Why secure?

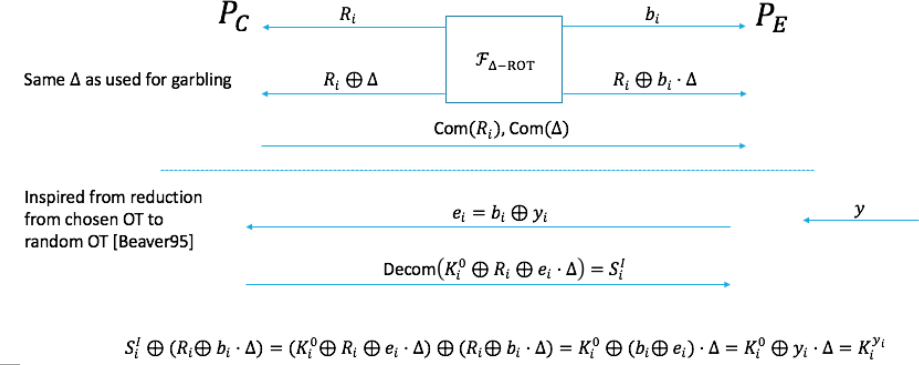


Intuitively,  $\text{Decom}(K_i^0 \oplus R_i \oplus e_i \cdot \Delta)$  is either correct for both, or neither input labels.

Wrong  $R_i$  commit:

- Com( $\widehat{R}_i$ ) where  $\widehat{R}_i \neq R_i$ , then  $S_i^I \oplus (R_i \oplus b_i \cdot \Delta) \notin \{K_i^0, K_i^1\}$ .
- Solution: Majority sized Input Authenticators, if  $S_i^I \oplus (R_i \oplus b_i \cdot \Delta) \notin \{K_i^0, K_i^1\}$  will reject and  $P_E$  can abort.

# Why secure?

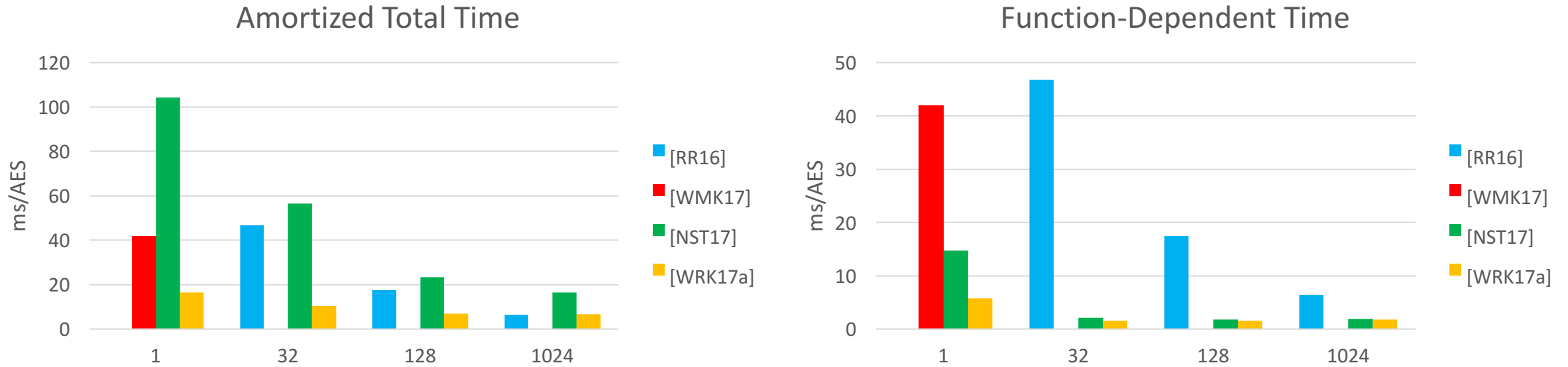


Intuitively,  $\text{Decom}(K_i^0 \oplus R_i \oplus e_i \cdot \Delta)$  is either correct for both, or neither input labels.

Flipped  $R_i$  commit:

- $\text{Com}(\widehat{R}_i)$  where  $\widehat{R}_i = R_i \oplus \Delta$ , then  $S_i^I \oplus (R_i \oplus b_i \cdot \Delta) = K_i^{1-y_i}$ .
- Solution: In preprocessing,  $P_C$  sends  $\text{Decom}(\text{lsb}(K_i^0))$ . Note:  $\text{lsb}(\Delta) = 1$ .

# Performance Comparison (AES-128)



AWS c4.8x instances, 10Gbit LAN. Excl Base OT cost.

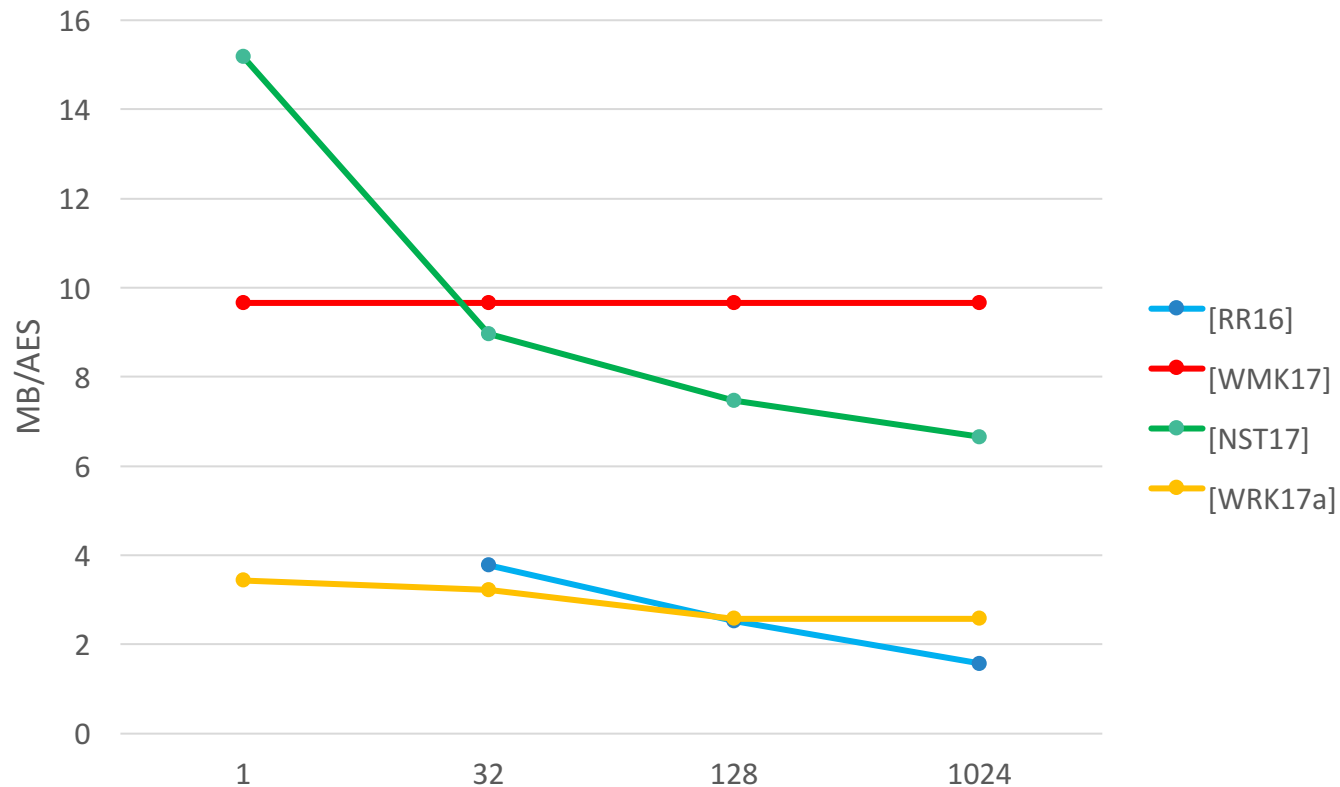
[RR16]: “Faster Malicious 2-party Secure Computation with Online/Offline Dual Execution”, USENIX 16.

[WMK17]: “Faster Two-Party Computation Secure Against Malicious Adversaries in the Single-Execution Setting”, Eurocrypt 17.

[WRK17a]: “Authenticated Garbling and Efficient Maliciously Secure Two-Party Computation”, ePrint 17.

# Observations

Amortized data sent from  $P_C$  to  $P_E$  per AES



N	[RR16]	[WMK17]	[NST17]	[WRK17a]
1	-	9,66	15,18	3,43
32	3,78	9,66	8,97	3,21
128	2,52	9,66	7,46	2,57
1024	1,58	9,66	6,66	2,57

LEGO has too high concrete communication complexity compared to other approaches.

Commitment cost dominate the preprocessing phase, ~70% of total time.

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# Cost of Malicious 2PC

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Two main costs of malicious 2PC based on Cut-and-Choose (C&C) of garbled circuits

- Replication factor, data communication *per gate*.
- Soldering cost, data communication *per input/output wire*.

Best implementations of two main approaches

- Whole-circuit C&C, e.g. [RR16, WMK17]
  - High replication factor  $O(s)$ , low soldering cost.
  - [RR16] amortizes over many executions of *same* function,  $O(s/\log(\#c))$ .
- Gate-level C&C, e.g. [NST17, ZH17, (WRK17a)]
  - Low replication factor, LEGO-factor  $O(s/\log(|C|))$ . [WRK17a] best constants.
  - High soldering cost,  $O(k|C|)$ , [NST17] best constants.



# Balls and Buckets [LR14]

Experiment:

- Let  $N$  be number of buckets,  $B$  the bucket size and  $p = 1/2$  is the checking probability.
- Adv  $\mathcal{A}$  prepares  $2NB$  balls and sends to Chal  $\mathcal{C}$ .
- $\mathcal{C}$  inspects  $NB$  balls at *random* and aborts if any are filthy.
- $\mathcal{C}$  packs the remaining  $NB$  balls at *random* into  $N$  buckets each of size  $B$ .
- $\mathcal{A}$  wins the game if any of the  $N$  buckets contain only filthy balls.

Theorem

$$\Pr[\mathcal{A} \text{ wins}] \leq \frac{\binom{2NB-t}{NB-t}}{\binom{2NB}{NB}} \cdot N \cdot \binom{t}{B} \cdot \binom{NB}{B}^{-1} \leq 2^{-t} \cdot N \cdot \binom{t}{B} \cdot \binom{NB}{B}^{-1}$$

where  $t$  is number of filthy balls.

As  $N$  increases, the bucket size  $B$  can be lowered accordingly.

Survive C&C

#Filthy subsets of size  $B$

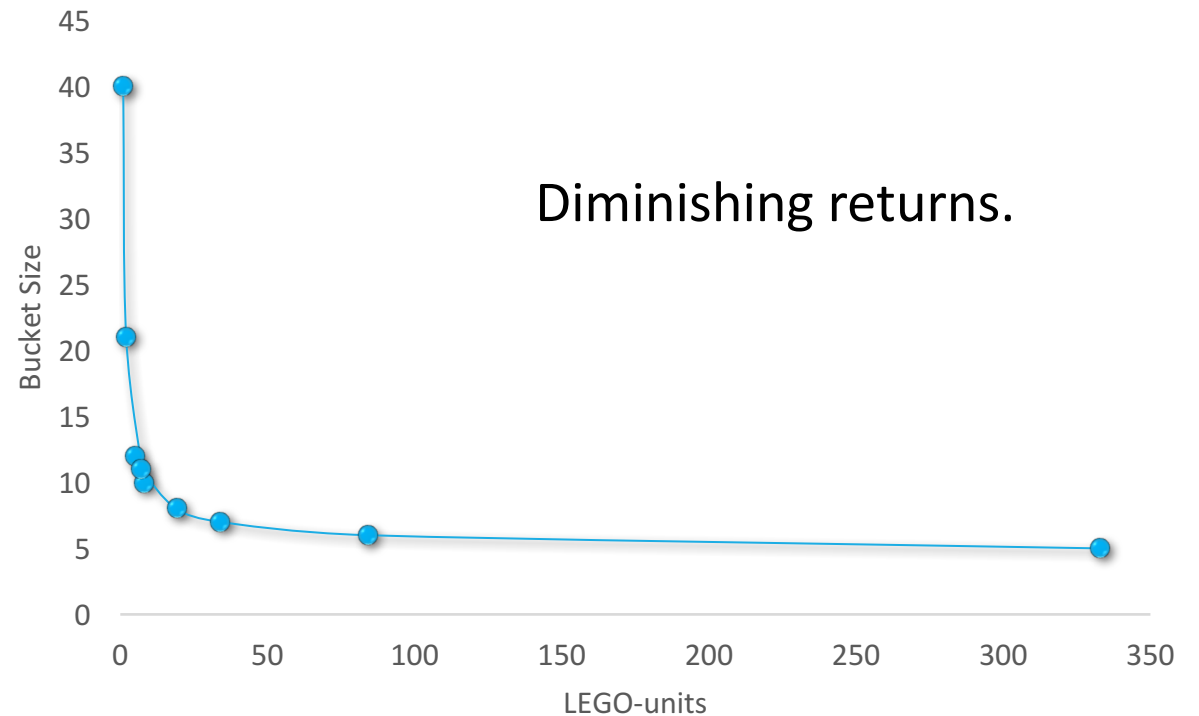
Union Bound

Prob. a bucket with  $B$  filthy balls.

# LEGO-factor

All mentioned protocols rely on the same “bucketing” analysis for replication factor  $O(s/\log(|C|))$

#LEGO-units (N)	Bucket Size
1	40
2	21
8	10
333	5
3,151	4
280,132	3
> 200,000,000,000	2



Check component prob.  $p = 1/2$ . Stat sec.  $s = 40$ .

# Introducing DUPLO

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Joint work with Vladimir Kolesnikov, Jesper Buus Nielsen, Mike Rosulek and Ni Trieu.

Propose LEGO style 2PC protocol with arbitrary component size

- Whole-circuit C&C has component size  $|C|$ .
- Gate C&C has component size 1.
- DUPLO supports *any size in this range*, also distinct component types.
- Similar idea considered in [GLMY16] for semi-honest setting to get function-independent preprocessing.

Fewer components than LEGO, but larger

- Low replication factor due to LEGO-factor diminishing returns.
- Lower commitment and soldering cost due to fewer overall input/output wires.
- Results in more efficient protocol, especially for programs comprised of many identical subcomponents (e.g. loops, arithmetic operations, subroutine calls).
- Implementation is up to 7x faster than [WMK17] and 5x faster than [RR16] for certain circuits.

# Scaling LEGO to larger components

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LEGO uses global free-XOR difference  $\Delta$

- To not leak during C&C, a gate is checked on a single input only. Only caught with prob.  $1/4$  ([ZH17] catches cheating with prob.  $1/2$ ).
- Does not scale to large input sizes, worst-case catch bad circuit with prob.  $1/2^n$  for n-input components.

DUPLO uses distinct  $\Delta_i$  free-XOR differences for each garbled component

- Allows to catch a checked component with prob. 1. Better concrete LEGO-factor.
- Can use variant of the C&C seed checking technique of [GMS08]. Check circuits much cheaper.

# Soldering with distinct differences

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Recall, with LEGO solderings are XOR of 0-label

- And  $P_C$  always commits to 0-label  $K^0$ , and the 1-label is defined as  $K^0 \oplus \Delta$ .

Consider garbled components  $G_i$  and  $G_j$  with distinct differences  $\Delta_i$  and  $\Delta_j$ .

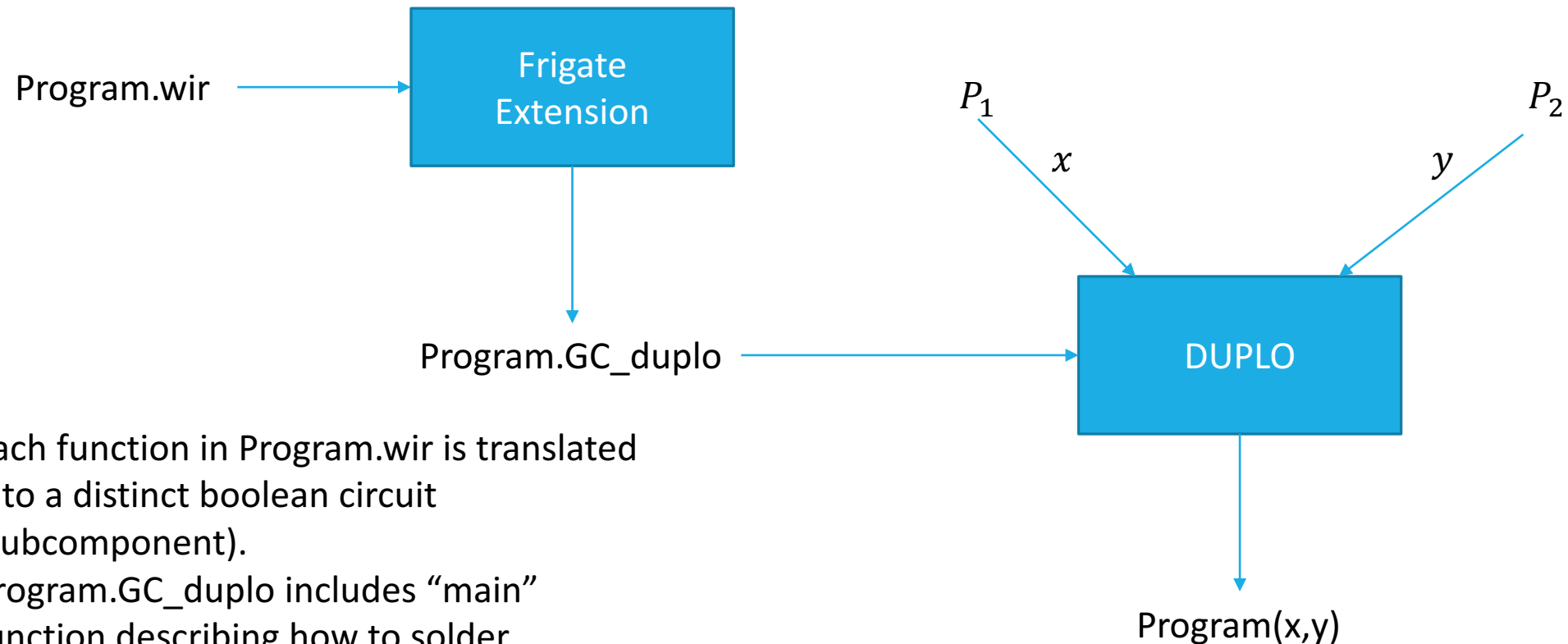
- No single value  $S^K$  can turn a label  $K_i^b = K_i^0 \oplus b \cdot \Delta_i$  into label  $K_j^b = K_j^0 \oplus b \cdot \Delta_j$ .

Use approach of [AHMR15] to solder circuits together

- XOR-homomorphic commitments to indicator bits.
- Essentially same cost as LEGO using various tricks and optimizations.

# A Tool for Program Decomposition

We extend the recent Frigate compiler [MGCBT16] to output circuits in a format suitable for DUPLO. Same input language as Frigate (C-like syntax).



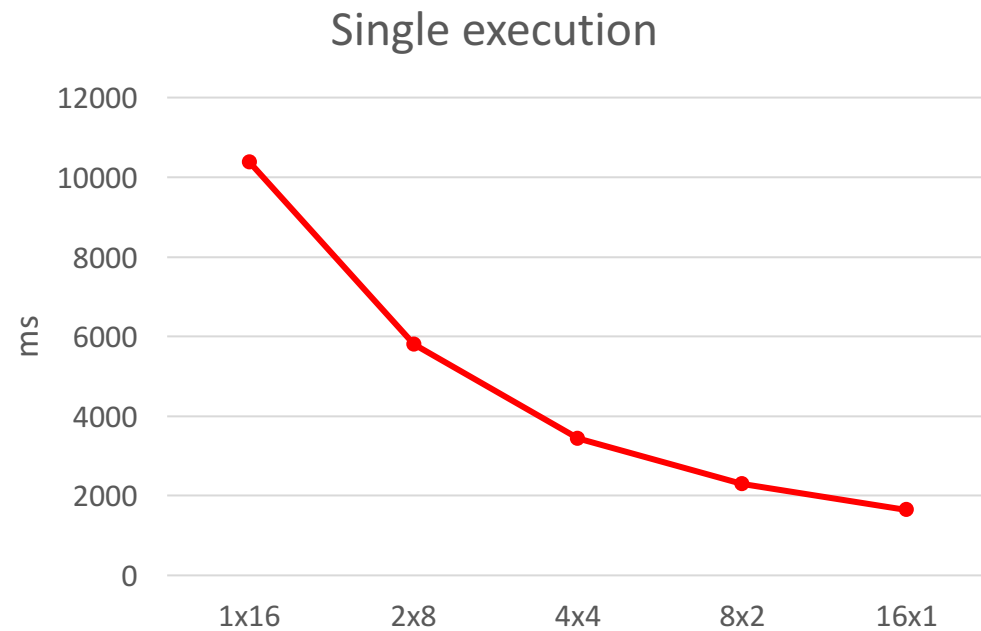
Each function in `Program.wir` is translated into a distinct boolean circuit (subcomponent).

`Program.GC_duplo` includes “main” function describing how to solder.

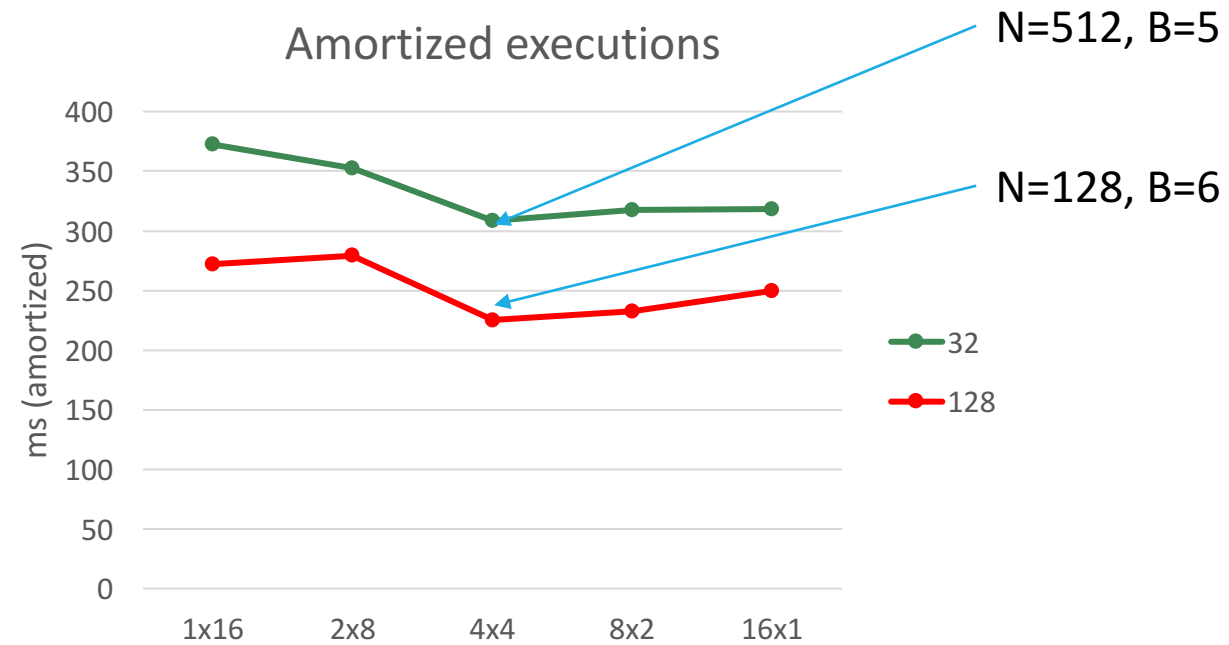
# AES-CBC-MAC-16 Decomposition

Contains 102,400 AND gates

- Can naturally be split into 1x16, 2x8, 4x4, 8x2, 16x1 AES subcomponents.



1 AES best subcomponent

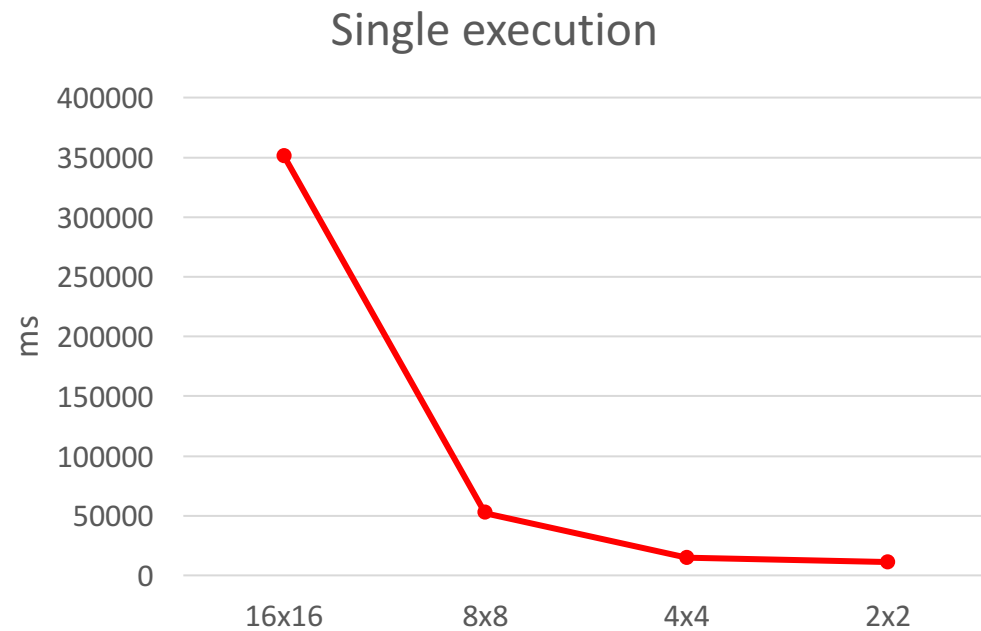


4 AES best subcomponent

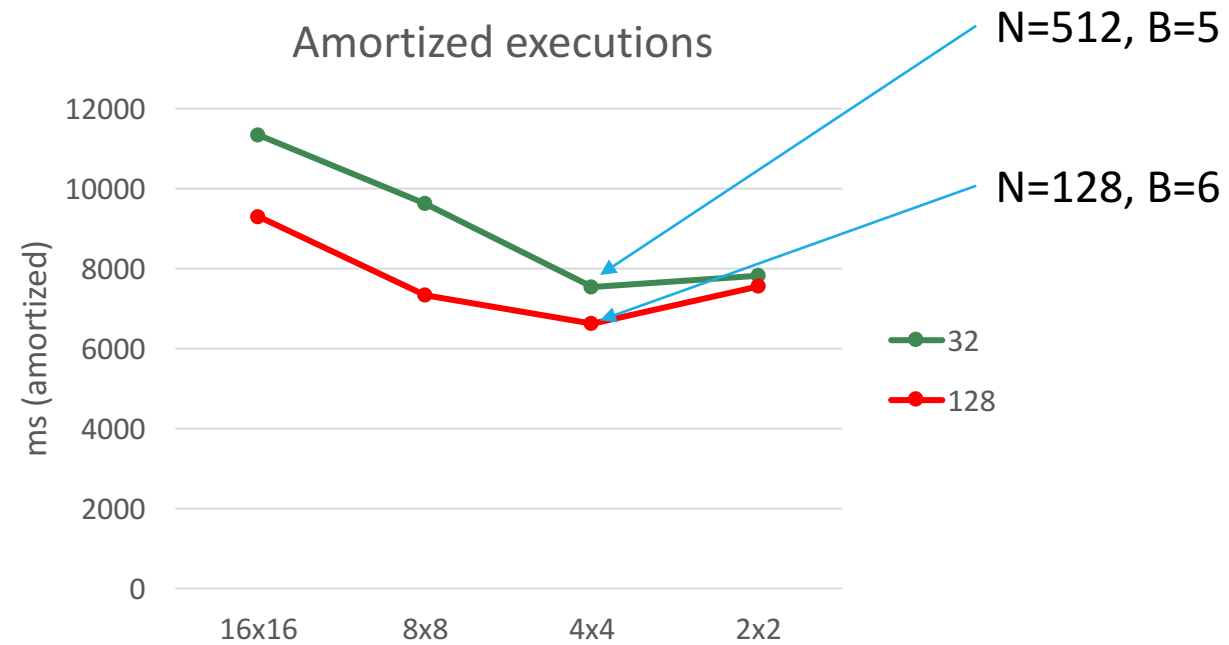
# 32-bit 16x16 Matrix Multiplication

Contains 4,194,304 AND gates

- Can split computation into  $m \times m$  32-bit matrix product and matrix addition blocks, for  $m = 2, 4, 8, 16$ .



2x2 block matrix best subcomponent

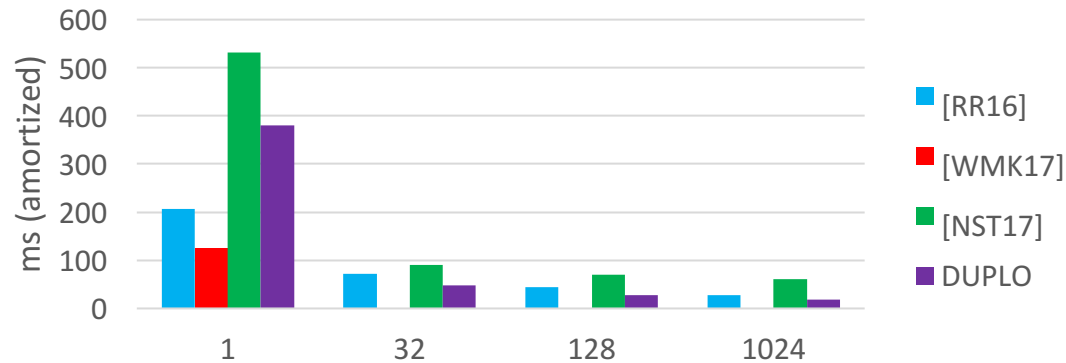


4x4 block matrix best subcomponent



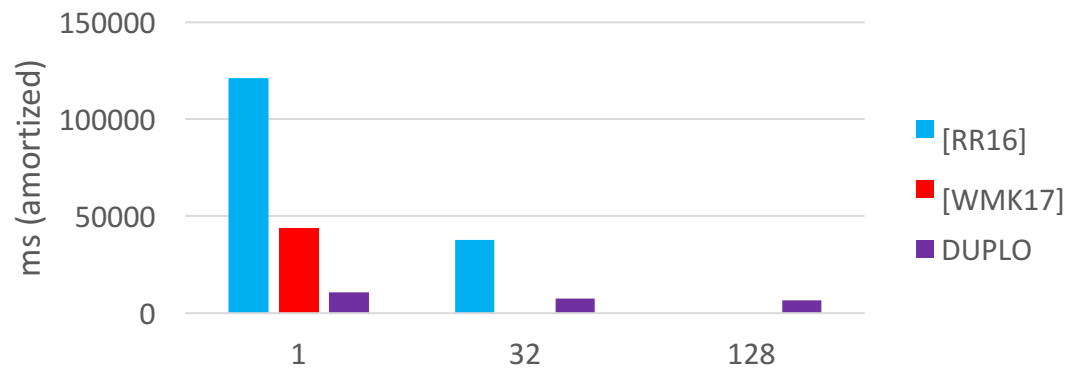
# Comparison (Same hardware, 1Gbit LAN)

Amortized AES (6,400 AND)



N	[RR16]	[WMK17]	[NST17]	DUPLO
1	207	125	531	380
32	71	-	90	48
128	44	-	70	28
1024	28	-	61	18

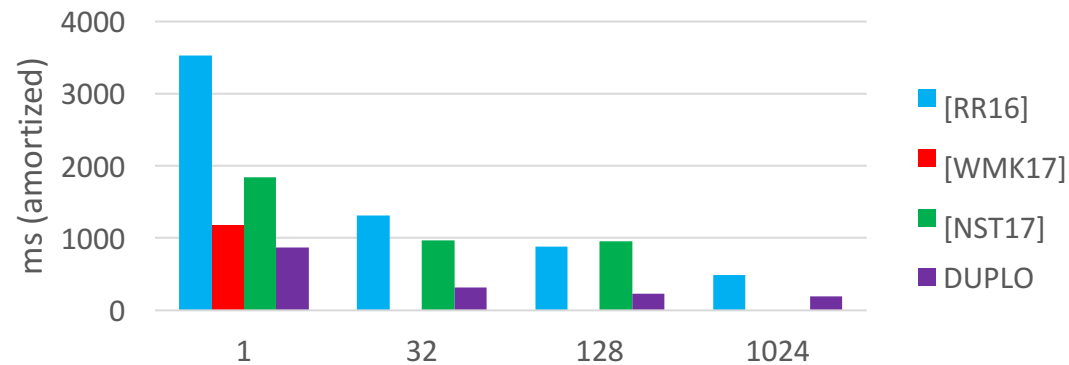
Amortized 32-bit MatMul (4,194,304 AND)



N	[RR16]	[WMK17]	DUPLO
1	121,113	43,930	10,955
32	37,684	-	7,539
128	Out of memory	-	6,622

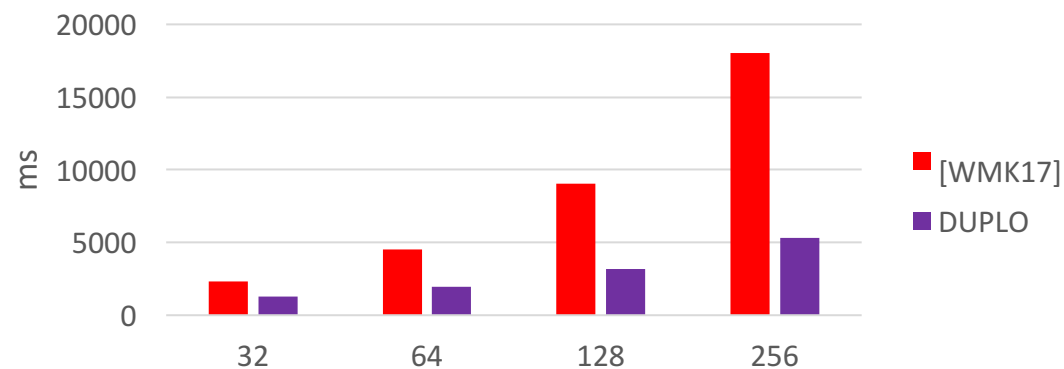
# Comparison (Same hardware, 1Gbit LAN)

Amortized AES-CBC-MAC-16 (102,400 AND)



N	[RR16]	[WMK17]	[NST17]	DUPLO
1	3,531	1,177	1,842	871
32	1,317	-	974	317
128	881	-	953	227
1024	488	-	Out of memory	190

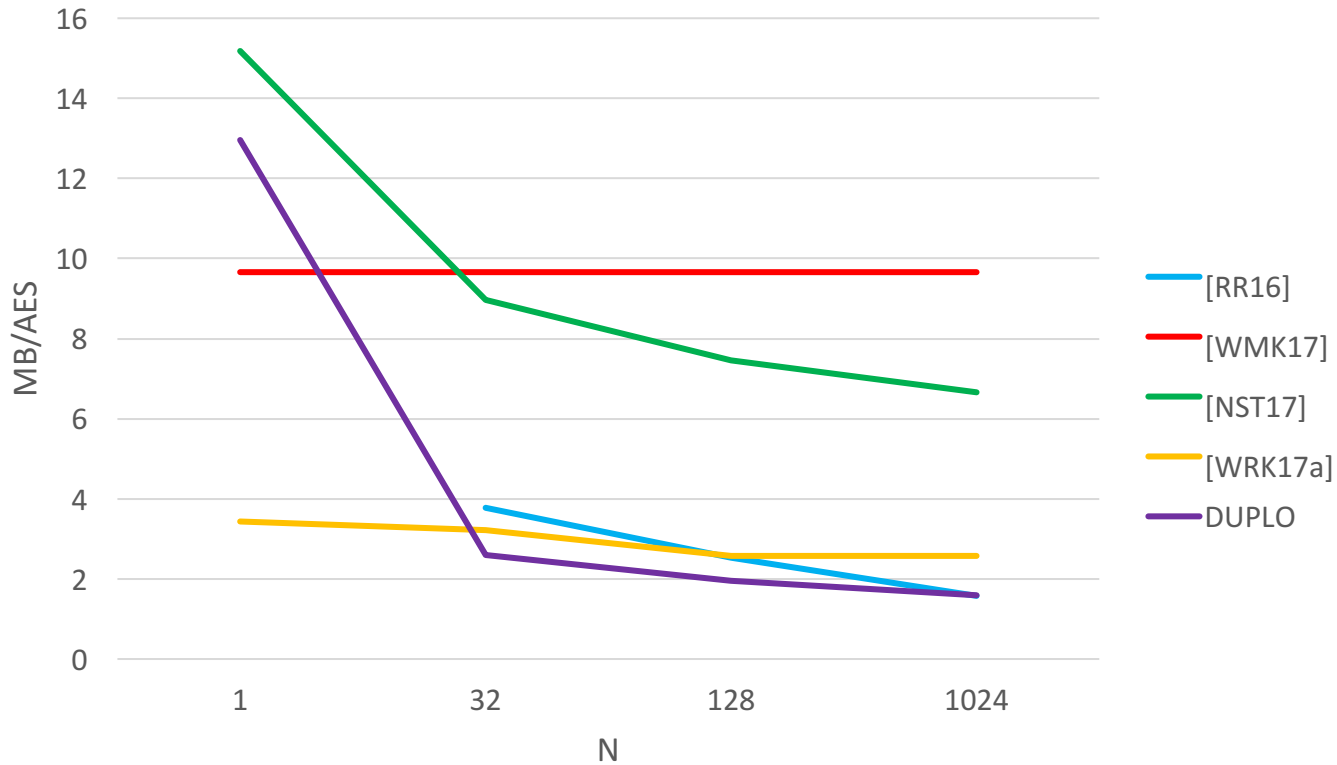
Single AES-CBC-MAC-N (6,400N AND)



N	[WMK17]	DUPLO
32	2,298	1,279
64	4,539	1,981
128	9,029	3,187
256	18,003	5,346

# Comparing Communication

Amortized data sent from  $P_C$  to  $P_E$  per AES



N	[RR16]	[WMK17]	[NST17]	[WRK17a]	DUPLO
1	-	9,66	15,18	3,43	12,96
32	3,78	9,66	8,97	3,21	2,60
128	2,52	9,66	7,46	2,57	1,96
1024	1,58	9,66	6,66	2,57	1,59

# Thank you

Questions?

Paper coming to ePrint “soon”

Source Code: <https://github.com/AarhusCrypto/DUPLO>

# References

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- [NST17] Jesper Buus Nielsen, Thomas Schneider, Roberto Trifiletti: **Constant Round Maliciously Secure 2PC with Function-independent Preprocessing using LEGO, NDSS 2017.**
- [BHR12] Mihir Bellare, Viet Tung Hoang, Phillip Rogaway: **Foundations of Garbled Circuits, CCS 2012.**
- [KS08] Vladimir Kolesnikov, Thomas Schnieder: **Improved Garbled Circuit: Free XOR Gates and Applications, ICALP 2008.**
- [ZRE15] Samee Zahur, Mike Rosulek, David Evans: **Two Halves Make a Whole: Reducing Data Transfer in Garbled Circuits using Half Gates, Eurocrypt 2015.**
- [Bra13] Luís T. A. N. Brandão: **Secure Two-Party Computation with Reusable Bit-Commitments, via a Cut-and-Choose with Forge-and-Lose Technique, Asiacrypt 2013.**
- [HKE13] Yan Huang, Jonathan Katz, David Evans: **Efficient Secure Two-Party Computation Using Symmetric Cut-and-Choose, CRYPTO 2013.**
- [Lin13] Yehuda Lindell: **Fast Cut-and-Choose Based Protocols for Malicious and Covert Adversaries, CRYPTO 2013.**
- [HKKKM14] Yan Huang, Jonathan Katz, Vladimir Kolesnikov, Ranjit Kumaresan, Alex J. Malozemoff : **Amortizing Garbled Circuits, CRYPTO 2014.**
- [LR14] Yehuda Lindell, Ben Riva: **Cut-and-Choose Based Two-Party Computation in the Online/Offline and Batch Settings, CRYPTO 2014.**
- [LR15] Yehuda Lindell, Ben Riva: **Blazing Fast 2PC in the Offline/Online Setting with Security for Malicious Adversaries, CCS 2015.**

# References

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- [RR16] Peter Rindal, Mike Rosulek: **Faster Malicious 2-Party Secure Computation with Online/Offline Dual Execution, USENIX 2016.**
- [NO16] Jesper Buus Nielsen, Claudio Orlandi: **Cross and Clean: Amortized Garbled Circuits with Constant Overhead, TCC 2016-B.**
- [NO09] Jesper Buus Nielsen, Claudio Orlandi: **LEGO for Two Party Secure Computation, TCC 2009.**
- [FJNNO13] Tore Kasper Frederiksen, Thomas P. Jakobsen, Jesper Buus Nielsen, Peter Sebastian Nordholt, Claudio Orlandi: **MiniLEGO: Efficient Secure Two-Party Computation From General Assumptions, Eurocrypt 2013.**
- [FJNT15] Tore Kasper Frederiksen, Thomas P. Jakobsen, Jesper Buus Nielsen, Roberto Trifiletti: **TinyLEGO: An Interactive Garbling Scheme for Maliciously Secure Two-party Computation, ePrint 2015.**
- [FJNT16] Tore Kasper Frederiksen, Thomas P. Jakobsen, Jesper Buus Nielsen, Roberto Trifiletti: **On the Complexity of Additively Homomorphic UC Commitments, TCC-A 2016.**
- [IKNP03] Yuval Ishai, Joe Kilian, Kobbi Nissim, Erez Petrank: **Extending Oblivious Transfers Efficiently, CRYPTO 2003.**
- [NNOB12] Jesper Buus Nielsen, Peter Sebastian Nordholt, Claudio Orlandi, Sai Sheshank Burra: **A New Approach to Practical Active-Secure Two-Party Computation, CRYPTO 2012.**
- [WRK17a] Xiao Wang, Samuel Ranellicci, Jonathan Katz: **Authenticated Garbling and Efficient Maliciously Secure Two-Party Computation, ePrint 2017.**
- [WRK17b] Xiao Wang, Samuel Ranellicci, Jonathan Katz: **Authenticated Garbling and Efficient Maliciously Secure Multi-Party Computation, ePrint 2017.**
- [HSV17] Carmit Hazay, Peter Scholl, Eduardo Soria-Vazquez: **Low Cost Constant Round MPC Combining BMR and Oblivious Transfer, ePrint 2017.**

# References

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- [MF06] Payman Mohassel, Matthew Franklin: **Efficiency Tradeoffs for Malicious Two-party Computation, PKC 2006.**
- [KS06] Mehmet S. Kiraz and Berry Schoenmakers: **A Protocol Issue for the Malicious Case of Yao's Garbled Circuit Construction, SITB 2006.**
- [LP07] Yehuda Lindell, Benny Pinkas: **An efficient protocol for secure two-party computation in the presence of malicious adversaries, Eurocrypt 2007.**
- [sS11] abhi shelat, Chih-Hao Shen: **Fast two-party secure computation with minimal assumptions, CCS 13**
- [Beaver95] Donald Beaver: **Precomputing Oblivious Transfer, CRYPTO 1995.**
- [WMK17] Xiao Wang, Alex J. Malozemoff, Jonathan Katz: **Faster Two-Party Computation Secure Against Malicious Adversaries in the Single-Execution Setting, Eurocrypt 2017.**
- [ZH17] Ruiyu Zhu, Yan Huang: **Faster LEGO-based Secure Computation without Homomorphic Commitments, ePrint 2017.**
- [GLMY16] Adam Groce, Alex Ledger, Alex J. Malozemoff, Arkady Yerukhimovich: **CompGC: Efficient Offline/Online Semi-honest Two-party Computation, ePrint 2016.**
- [GMS08] Vipul Goyal, Payman Mohassel, Adam Smith: **Efficient Two Party and Multi Party Computation Against Covert Adversaries, Eurocrypt 2008.**
- [AHMR15] Arash Afshar, Zhangxiang Hu, Payman Mohassel, Mike Rosulek: **How to Efficiently Evaluate RAM Programs with Malicious Security, Eurocrypt 2015.**
- [MGCBT16] Benjamin Mood, Debayan Gupta, Henry Carter, Kevin R. B. Butler, Patrick Traynor: **Frigate: A Validated, Extensible, and Efficient Compiler and Interpreter for Secure Computation, EuroS&P 2016.**