Flood-It

The Colourful Game of Board Domination

Bristol Algorithms Day, 15–16 Feb 2009

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The Game
The Game
The Game
The Game
The Game
The Game
The Game
The Game
The Game
The Game

Done!

10 moves
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
Greedy
"Well done"

\( n \) moves
Greedy
Greedy
Greedy

Much better!

3 moves
2 Colours
2 Colours
2 Colours
2 Colours
2 Colours
2 Colours
2 Colours
Shortest Common Supersequence

1. a a b a c d c
2. c a b b a
3. b a d d d c a
4. a d d c a b
5. b a d c c d a a
6. d c a a b d c
7. c a b a d
8. b c d a a b c
Shortest Common Supersequence

1. a a b a c d c
2. c a b b a
3. b a d d d c a
4. a d d c a b
5. b a d c c d a a
6. d c a a b d c
7. c a b a d
8. b c d a a b c

a a b c a d c a d a c b c a b d c a a
Shortest Common Supersequence

1. $a$ $a$ $b$ $a$ $c$ $d$ $c$
2. $c$ $a$ $b$ $b$ $a$
3. $b$ $a$ $d$ $d$ $d$ $c$ $a$
4. $a$ $d$ $d$ $c$ $a$ $b$
5. $b$ $a$ $d$ $c$ $c$ $d$ $a$ $a$
6. $d$ $c$ $a$ $a$ $b$ $d$ $c$
7. $c$ $a$ $b$ $a$ $d$
8. $b$ $c$ $d$ $a$ $a$ $b$ $c$

$a$ $a$ $b$ $c$ $a$ $d$ $c$ $a$ $d$ $a$ $c$ $b$ $c$ $a$ $b$ $d$ $c$ $a$ $a$
Shortest Common Supersequence

1. $a$ $a$ $b$ $a$ $c$ $d$ $c$
2. $c$ $a$ $b$ $b$ $a$

- $\textbf{NP}$-hard, even with a binary alphabet,
- no polynomial-time constant factor approximation algorithm, unless $\textbf{P} = \textbf{NP}$. 

6. $d$ $c$ $a$ $a$ $b$ $d$ $a$ $c$

7. $c$ $a$ $b$ $a$ $d$

8. $b$ $c$ $d$ $a$ $a$ $b$ $c$

$\begin{align*}
\quad a & \quad a & \quad b & \quad c & \quad a & \quad d & \quad c & \quad a & \quad d & \quad a & \quad c & \quad b & \quad c & \quad a & \quad b & \quad d & \quad c & \quad a & \quad a \\
\end{align*}$
4 or More Colours

\[
\begin{array}{cccc}
  a & b & b & a \\
  a & b & & \\
\end{array}
\]
4 or More Colours

a  b  b  a  a  b
4 or More Colours

\[
\begin{array}{cccc}
    a & b & b & a \\
\end{array}
\]
4 or More Colours

\[ a \, b \, b \, a \quad a \, b \]
4 or More Colours

\[ a \quad b \quad b \quad a \quad a \quad b \]
4 or More Colours

\[ a \ b \ b \ a \]

[Diagram of a 4x4 grid with a highlighted square]
4 or More Colours

\[
\begin{array}{cccc}
a & b & b & a \\
\end{array}
\]

\[
\begin{array}{cc}
a & b \\
\end{array}
\]
4 or More Colours

\[ a \ b \ b \ a \]

\[ a \ b \]
4 or More Colours

\[ a \ b \ b \ a \]

\[ a \ b \]
4 or More Colours

\[ a \ b \ b \ a \]

\[ a \ b \]
4 or More Colours

\[
\begin{array}{cccc}
a & b & b & a \\
\end{array}
\]
4 or More Colours

\[ a \quad b \quad b \quad a \quad a \quad b \]
4 or More Colours
4 or More Colours

\[ \begin{array}{cccc}
  a & b & b & a \\
  a & b \\
\end{array} \]
4 or More Colours
4 or More Colours

\[ \text{Diagram showing a sequence of colors: } a \ b \ b \ a \ \text{and } a \ b \]
4 or More Colours
3 Colours

a b b a

a b
3 Colours

Is there a common supersequence of length at most 4?
3 Colours
3 Colours

[Diagram showing a grid with different colored squares and a sequence of 'a', 'b', 'b', 'a', 'a', 'b'].

3 Colours
3 Colours
3 Colours
3 Colours

a b b a

a b

1 2 3 4 5 6 7 8 10
Approximation
Approximation

and so on...
Approximation

\[(c - 1)\text{-approximation}\]

and so on…
Randomised Approximation

Flooding sequence: 

Red, blue, ..., orange.
Randomised Approximation

Flooding sequence: 

Shuffle
Randomised Approximation

Flooding sequence:

Shuffle
Randomised Approximation

Flooding sequence:

Probability $\frac{1}{2}$

Shuffle
Randomised Approximation

Flooding sequence:

Probability $\frac{1}{2}$

Shuffle
Randomised Approximation

Flooding sequence: 🟡 🟢 ...

Randomised \((2c/3)\)-approximation

Shuffle
Bad Boards
Bad Boards

Requires

$\Omega(n \sqrt{c})$

number of moves
Upper Bound
Upper Bound
Upper Bound
Upper Bound
Upper Bound

$n - 1$
Upper Bound

\[ \frac{x}{2} \]
Upper Bound

\[ n - 1 \]

\[ \frac{x}{2} \]
Upper Bound

\[ x \leq \frac{n - 1}{2} \]
Upper Bound

\[(n - 1) \cdot \left(\frac{n}{x} - 1\right) \cdot (n - 1)\]
Upper Bound

\[
\frac{x}{2} \quad x \quad x \quad x \quad \frac{x}{2}
\]

\[(\frac{n}{x} - 1) (n - 1)\]

\[c (n x - 1) (n - 1)\]
Upper Bound

\[
\left( \frac{n}{x} - 1 \right) (n - 1)
\]
Upper Bound

\[ \left( \frac{n}{x} - 1 \right) (n - 1) \]

\[ \frac{x}{2} C \]
Upper Bound

At most \( O(n \sqrt{c}) \) moves required

\[
\left( \frac{n}{x} - 1 \right) (n - 1)
\]
Upper Bound

At most

\[ O(n\sqrt{c}) \]

moves required

\[ \left( \frac{n}{x} - 1 \right) (n - 1) \]
Random Boards
Random Boards

$m$ moves to flood the board
Random Boards

$m$ moves to flood the board
Random Boards

$m$ moves to flood the board

At most $m$ colour changes
Random Boards

\(m\) moves to flood the board

At most \(m\) colour changes
Random Boards

$m$ moves to flood the board

At most $m$ colour changes
Random Boards

$m$ moves to flood the board

Also at most $m$ colour changes

At most $m$ colour changes
We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most $k$ colours changes. The bound depends on $k$, number of colours $c$ and the length of the path.
Random Boards

We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most $k$ colours changes. The bound depends on $k$, number of colours $c$ and the length of the path.

The probability that there exists any non-touching path from the top left to the bottom right tile with at most $k$ colour changes is upper bounded by the union bound over all non-touching paths from the top left to the bottom right tile.
We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most \( k \) colours changes. The bound depends on \( k \), number of colours \( c \) and the length of the path.

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This union bound is an upper bound on the probability that the board is flooded within \( k \) moves.
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Random Boards

We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most $k$ colours changes. The bound depends on $k$, number of colours $c$ and the length of the path.

$k$ is order $n$

The probability that there exists any non-touching path from the top left to the bottom right tile with at most $k$ colour changes is upper bounded by the union bound over all non-touching paths from the top left to the bottom right tile.

Probability less than $e^{-\Omega(n)}$ (for 3 or more colours)

This union bound is an upper bound on the probability that the board is flooded within $k$ moves.
Random Boards

We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most $k$ colours changes. The bound depends on $k$, number of colours $c$ and the length of the path.

Conclusion

The number of moves required to flood a random board is $\Omega(n)$ with high probability.

This union bound is an upper bound on the probability that the board is flooded within $k$ moves.

Probability less than $e^{-\Omega(n)}$ (for 3 or more colours)
Thank You!

Don’t forget our website:

http://floodit.cs.bris.ac.uk/
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