

FLOOD-IT

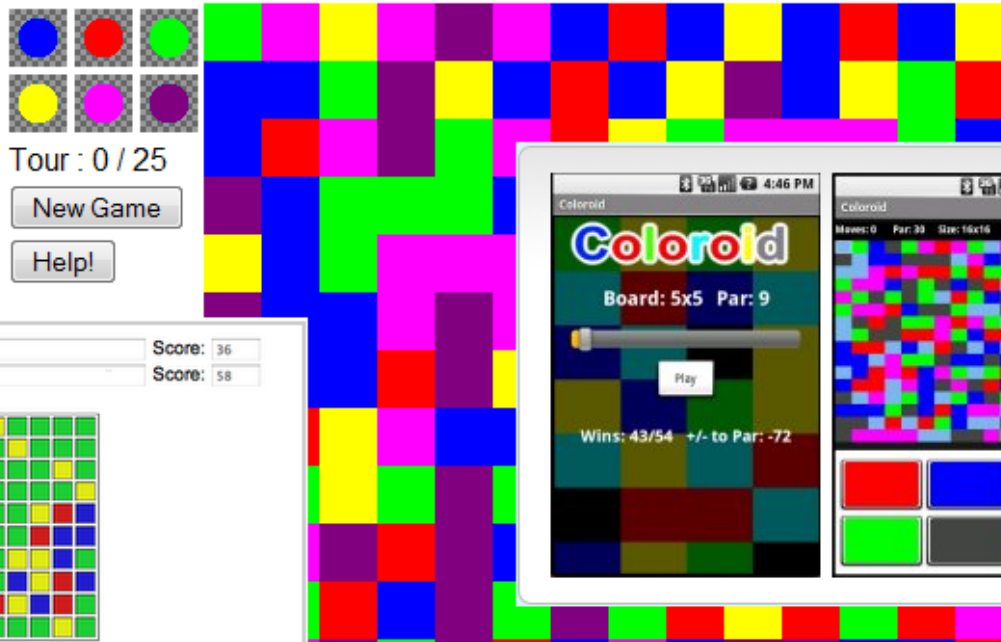
THE COLOURFUL GAME OF BOARD DOMINATION

Bristol Algorithms Day, 15–16 Feb 2009

David Arthur, Raphaël Clifford, Markus Jalsenius,
Ashley Montanaro and Benjamin Sach

Department of Computer Science





Tour : 0 / 25

New Game

Help!

Add Player 1 Score: 36
Add Player 2 Score: 58
Reset

Red Green Blue Yellow

Waiting for other player...

Coloroid
Board: 5x5 Par: 9
Play
Wins: 43/54 +/- to Par: -72

Coloroid
Moves: 0 Par: 30 Size: 16x16 Overall wins: 43/54

Coloroid
Score: 34 Par: 30 Size: 16x16 Overall wins: 43/54

Step 23/35
New Game

Flood-It!

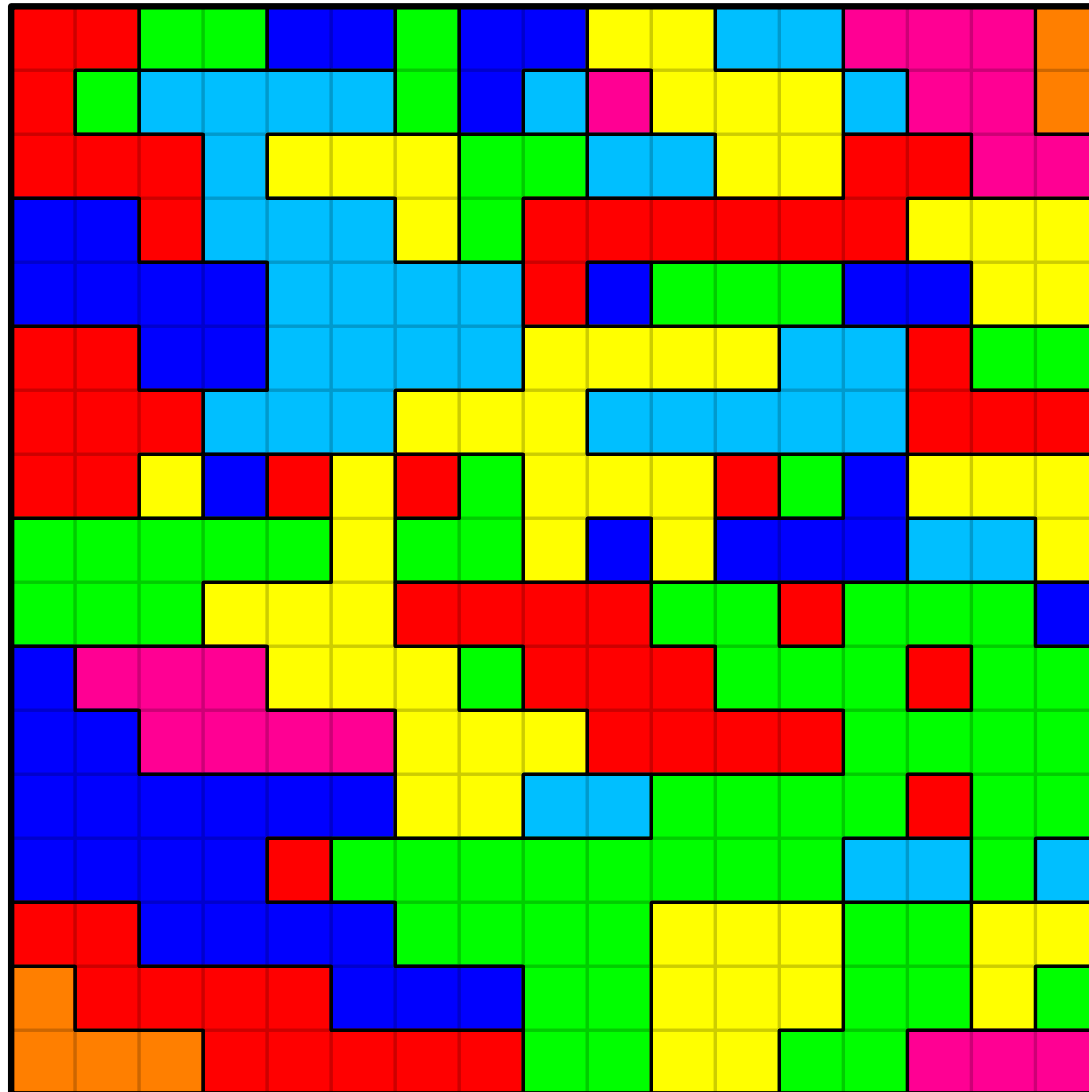
Use Flood-It! On



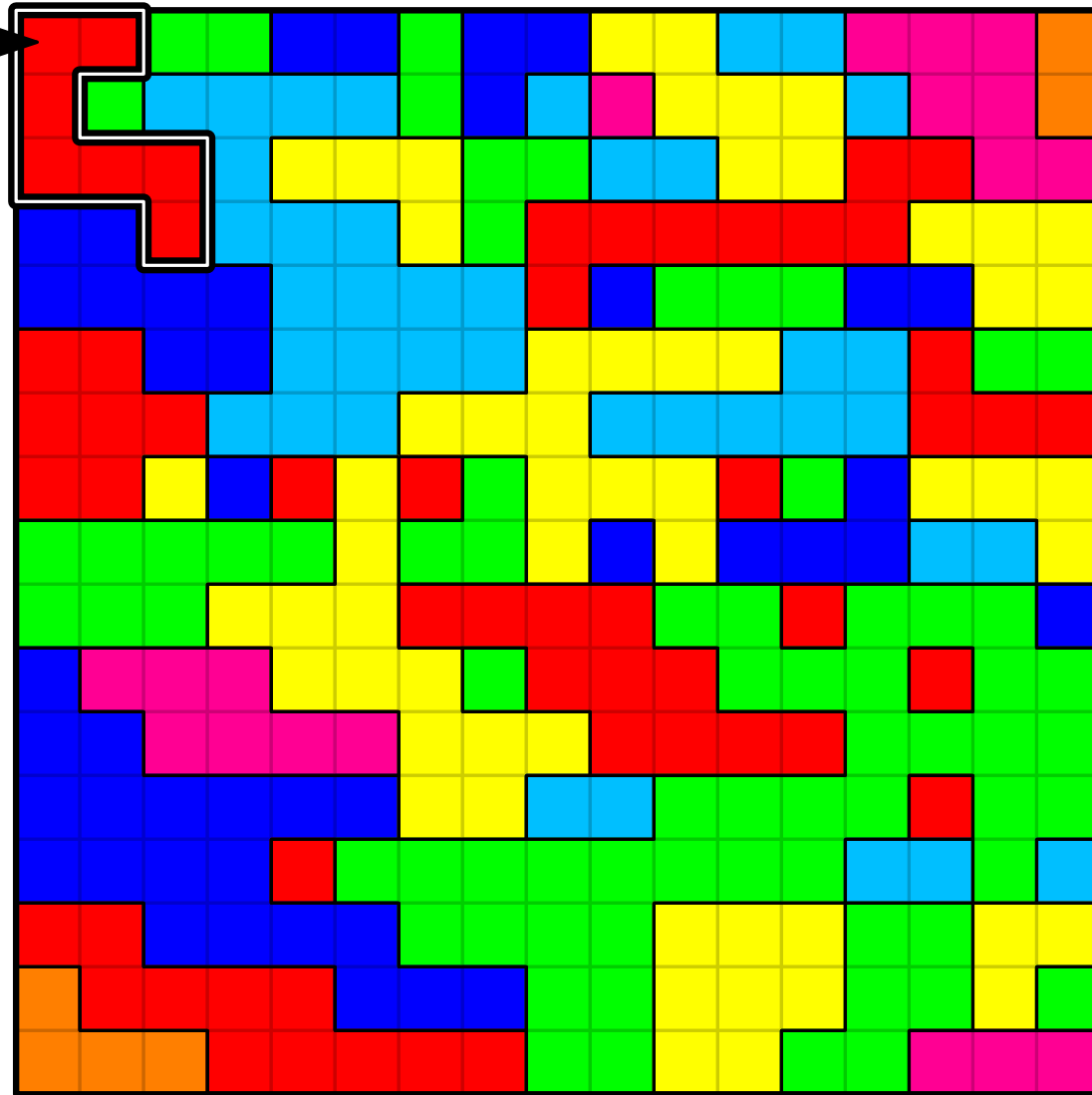
or try on other platforms:



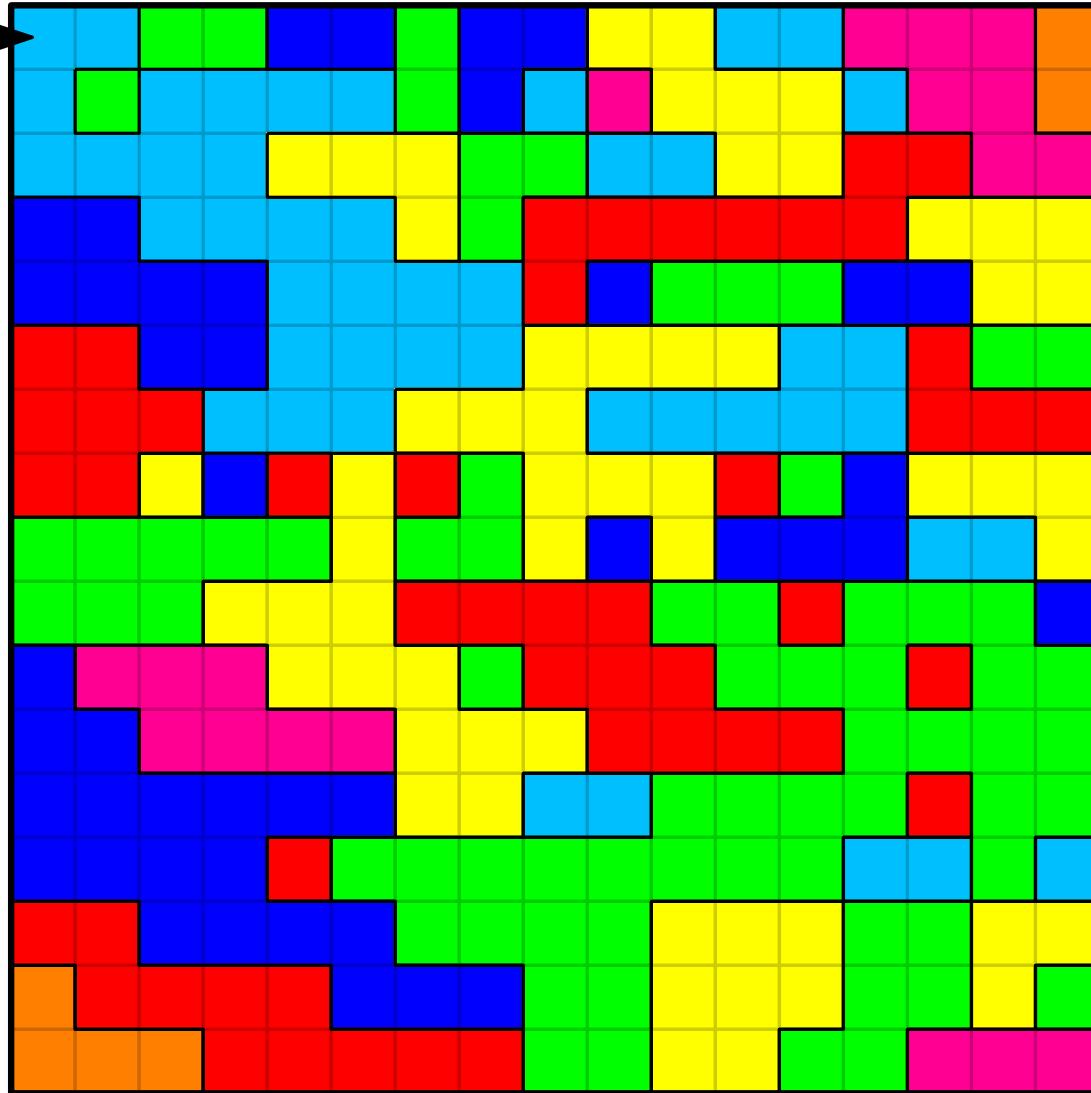
The Game



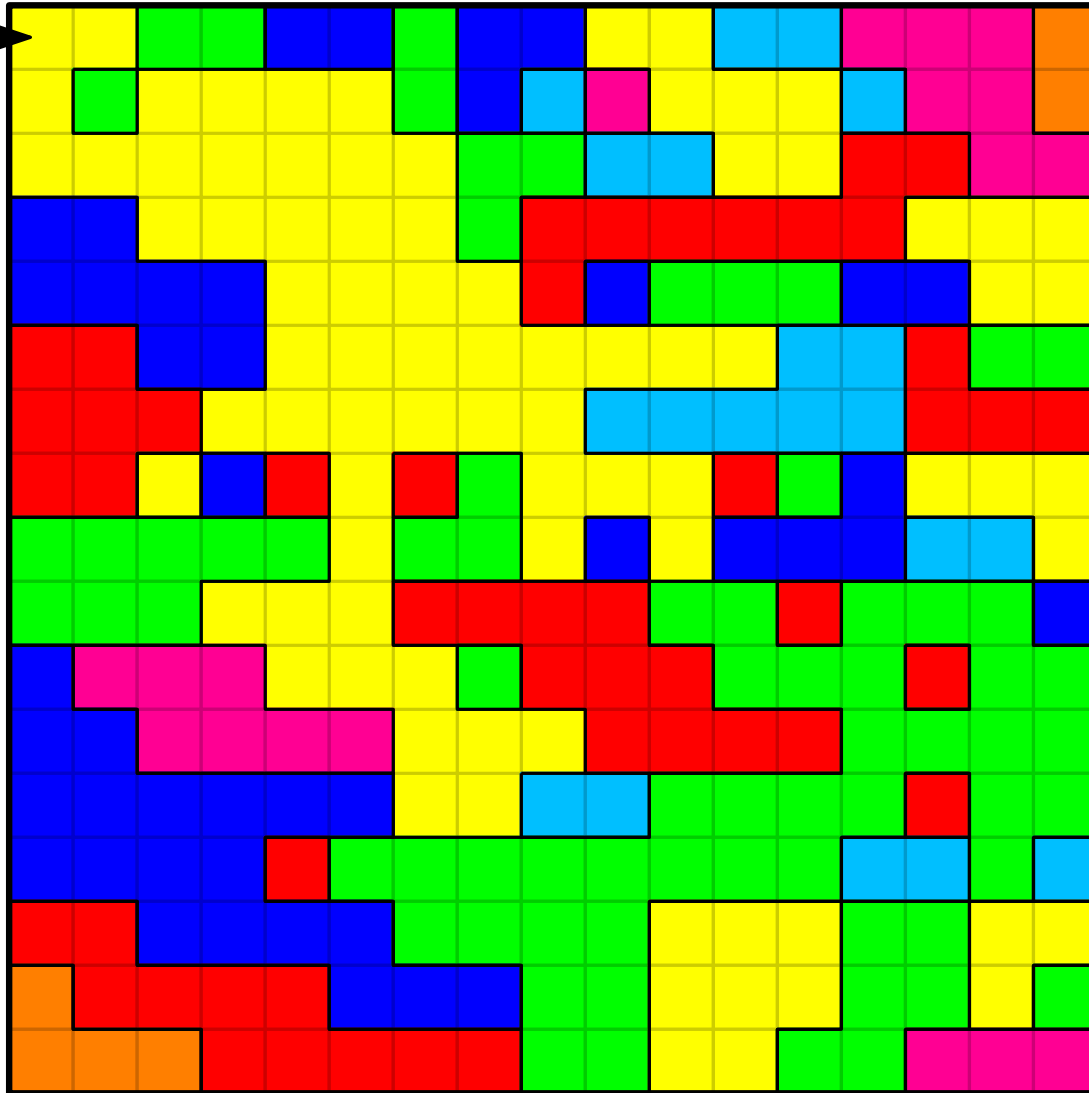
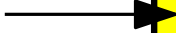
The Game



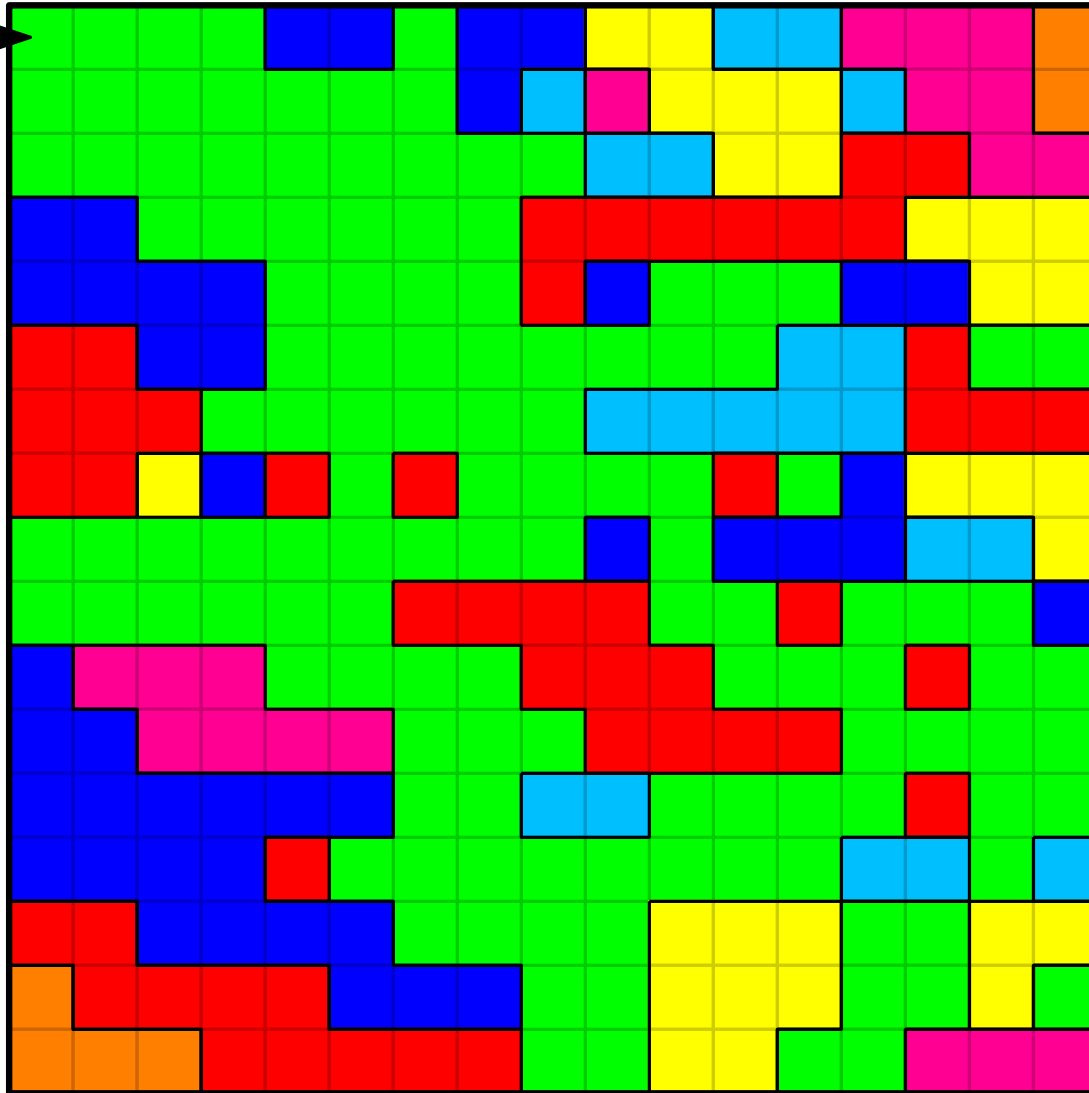
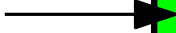
The Game



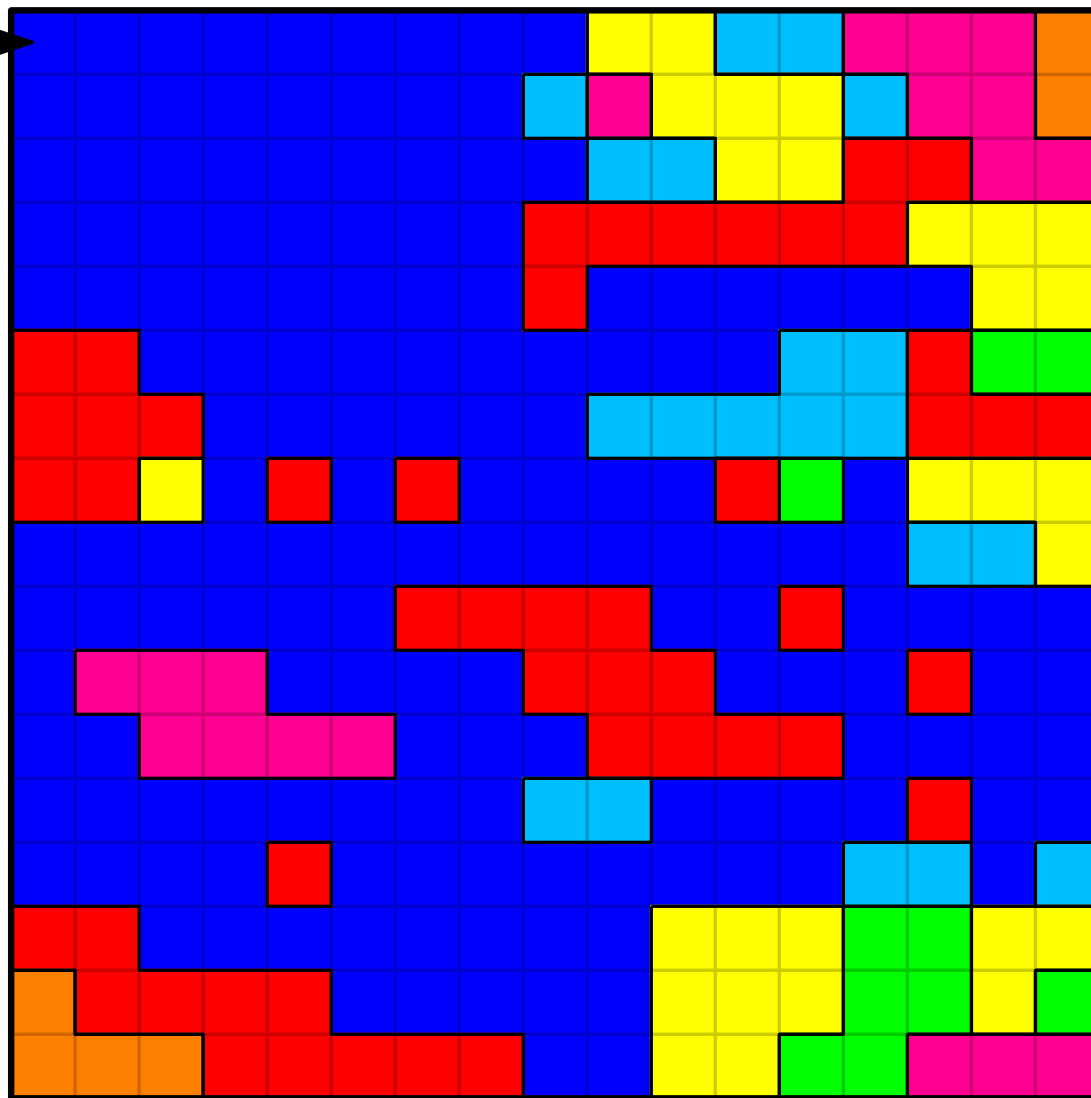
The Game



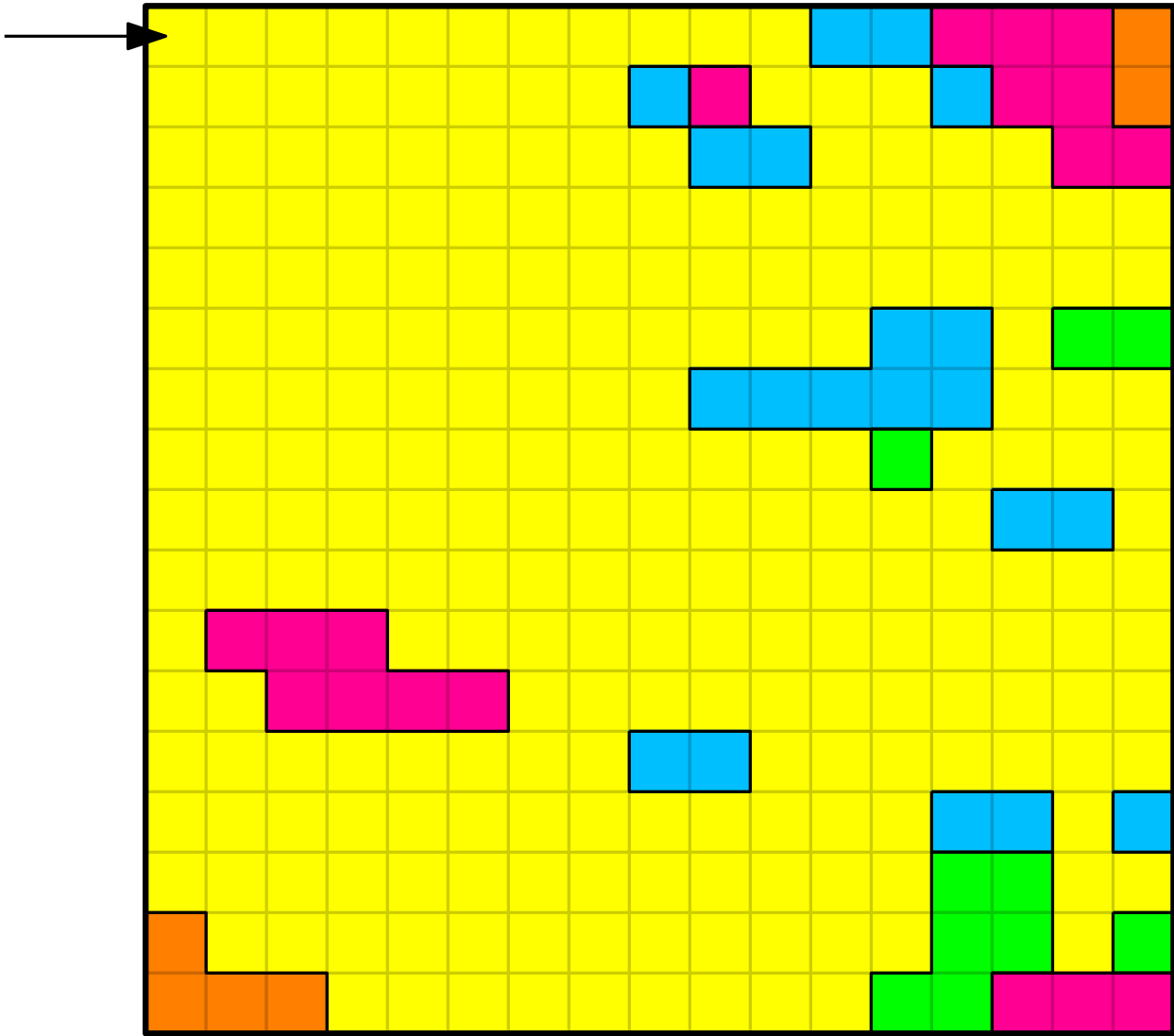
The Game



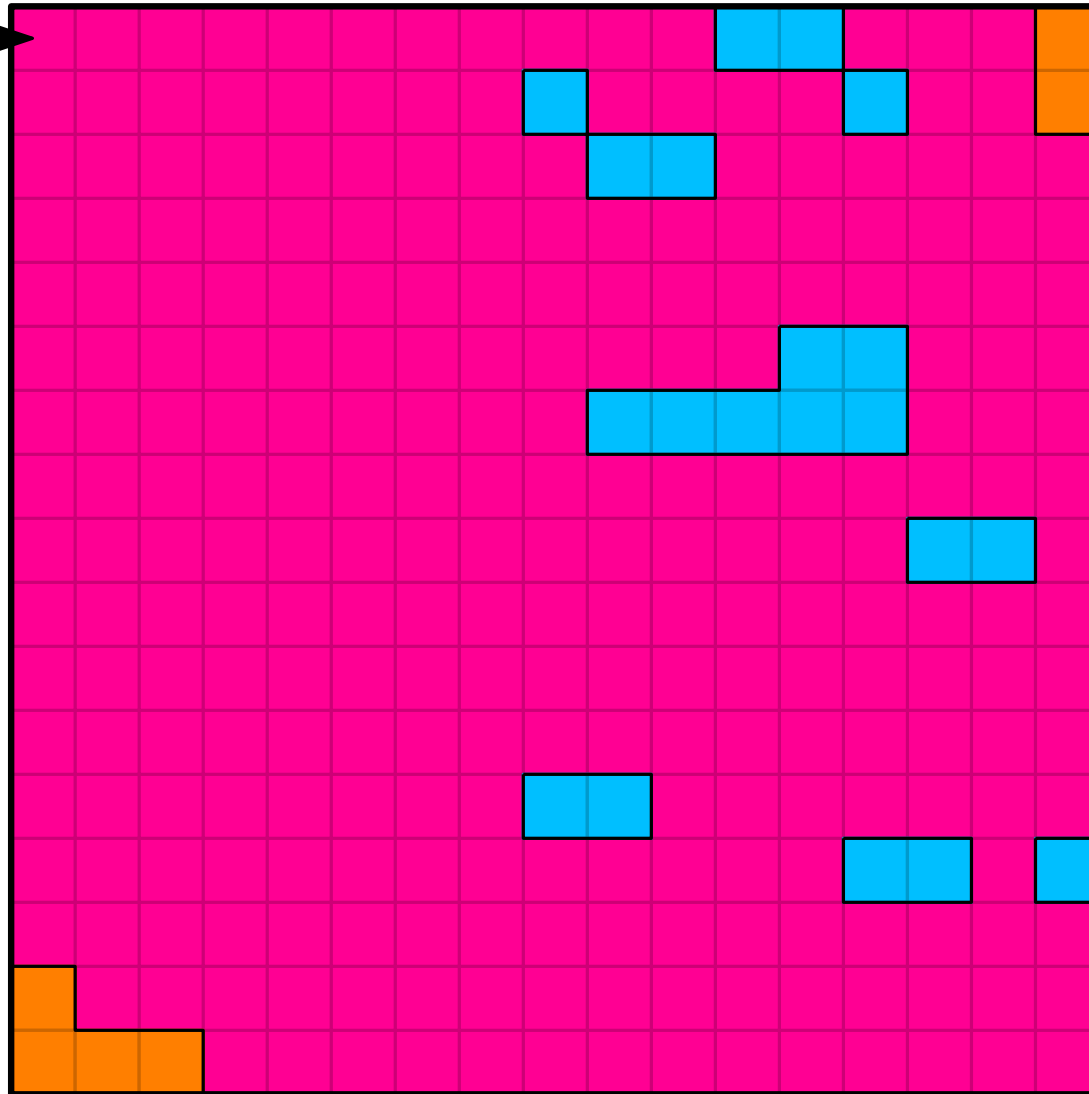
The Game



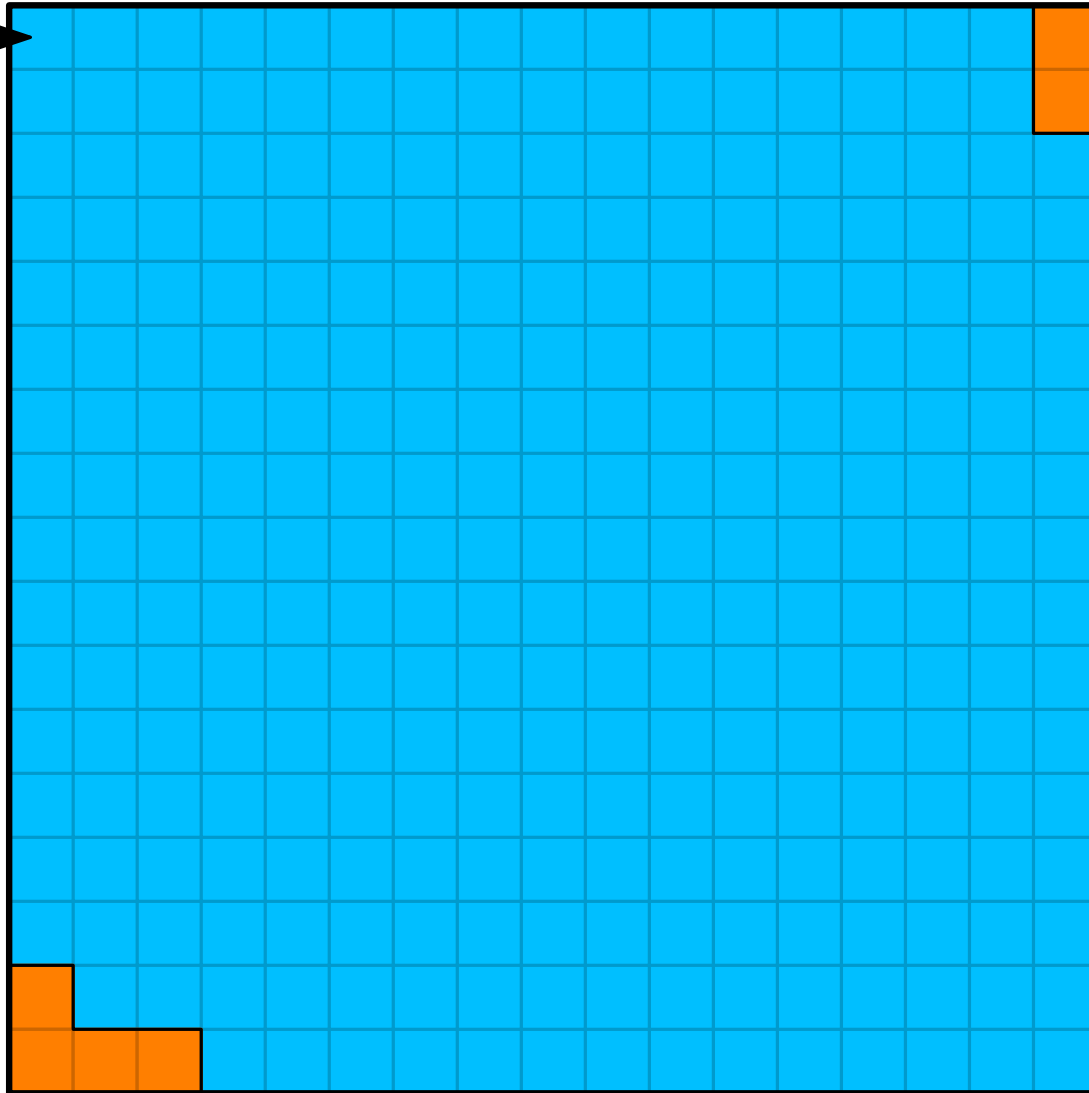
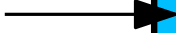
The Game



The Game



The Game

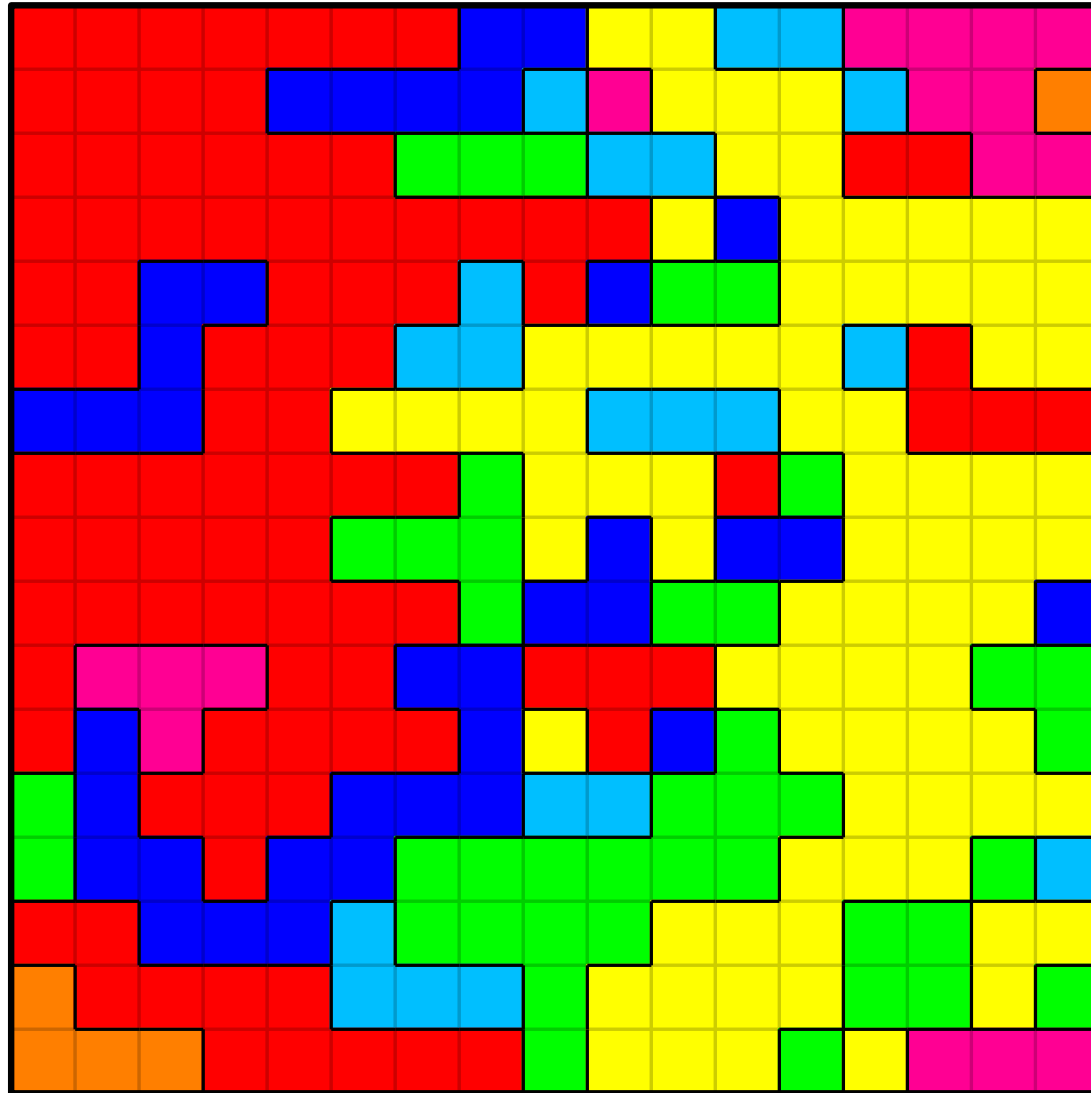


The Game

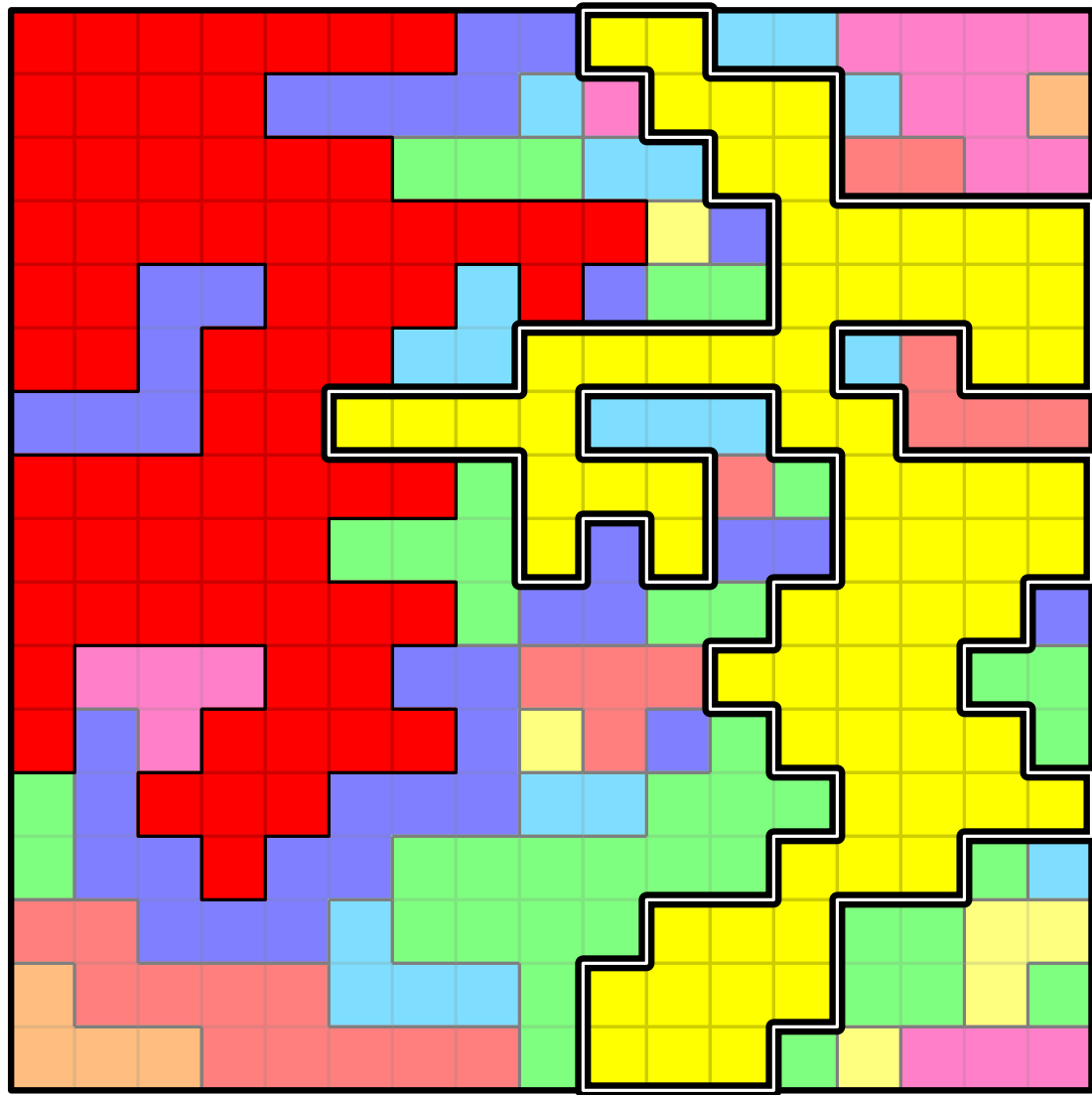
Done!

10 moves

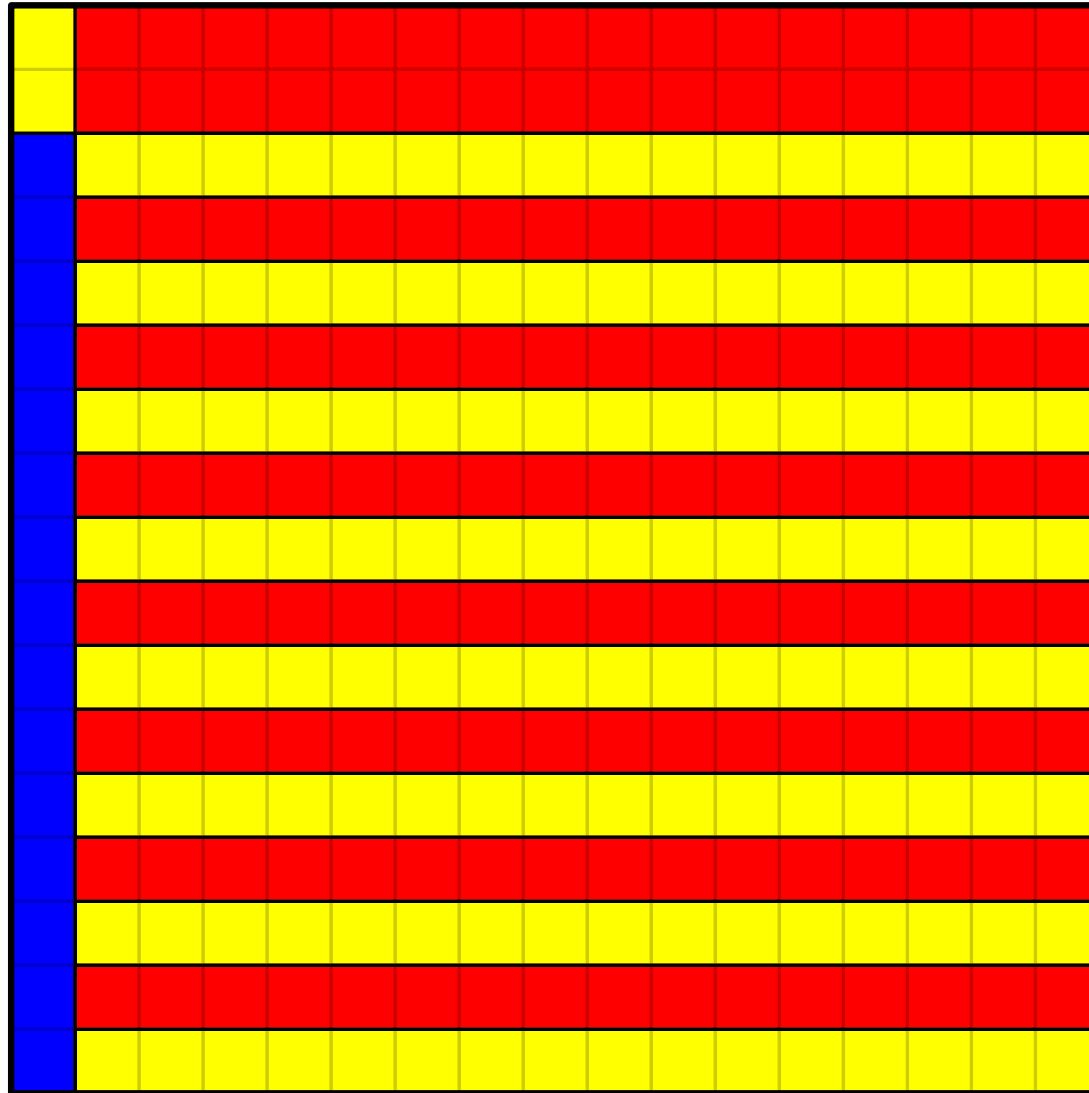
Greedy



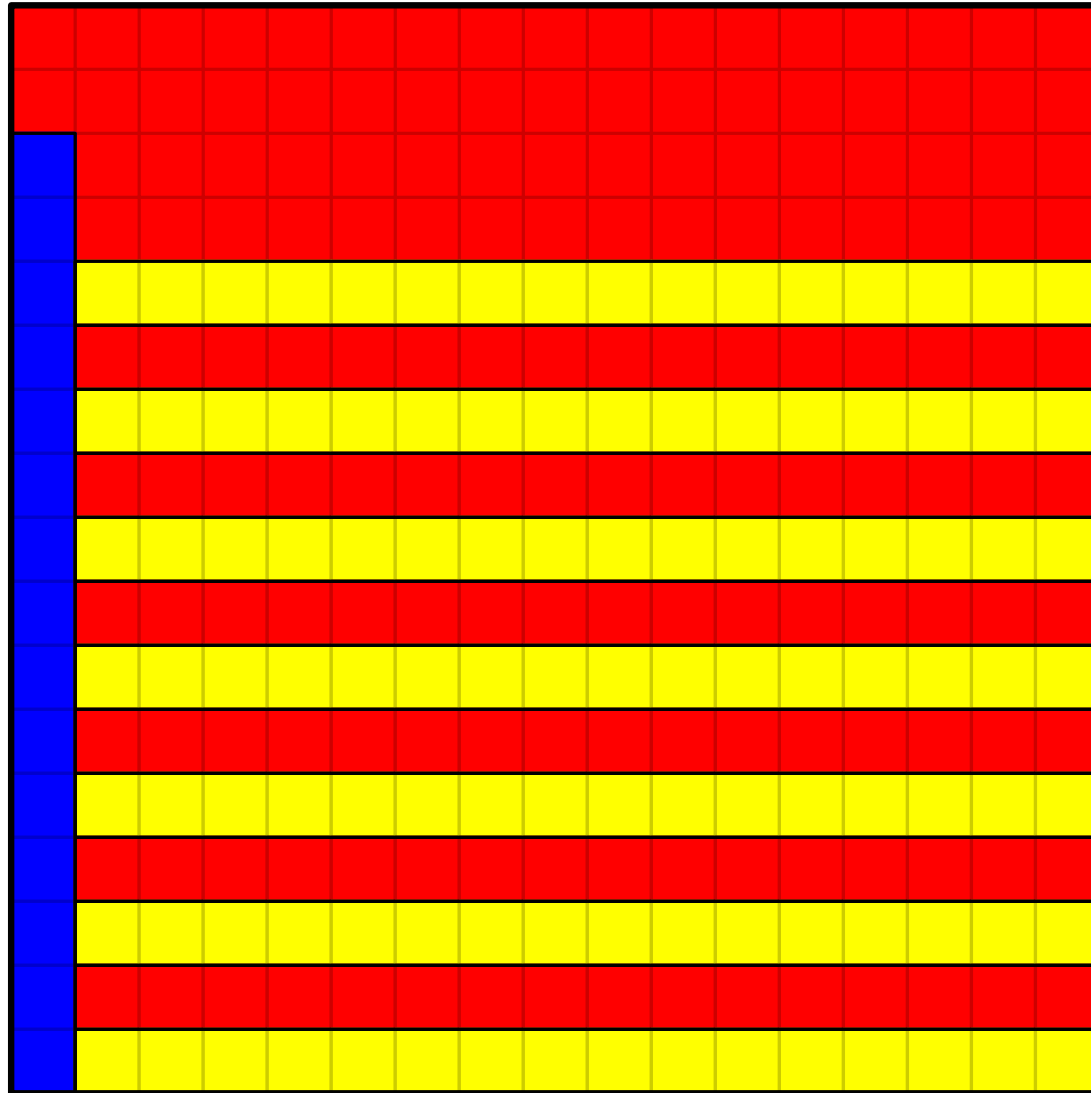
Greedy



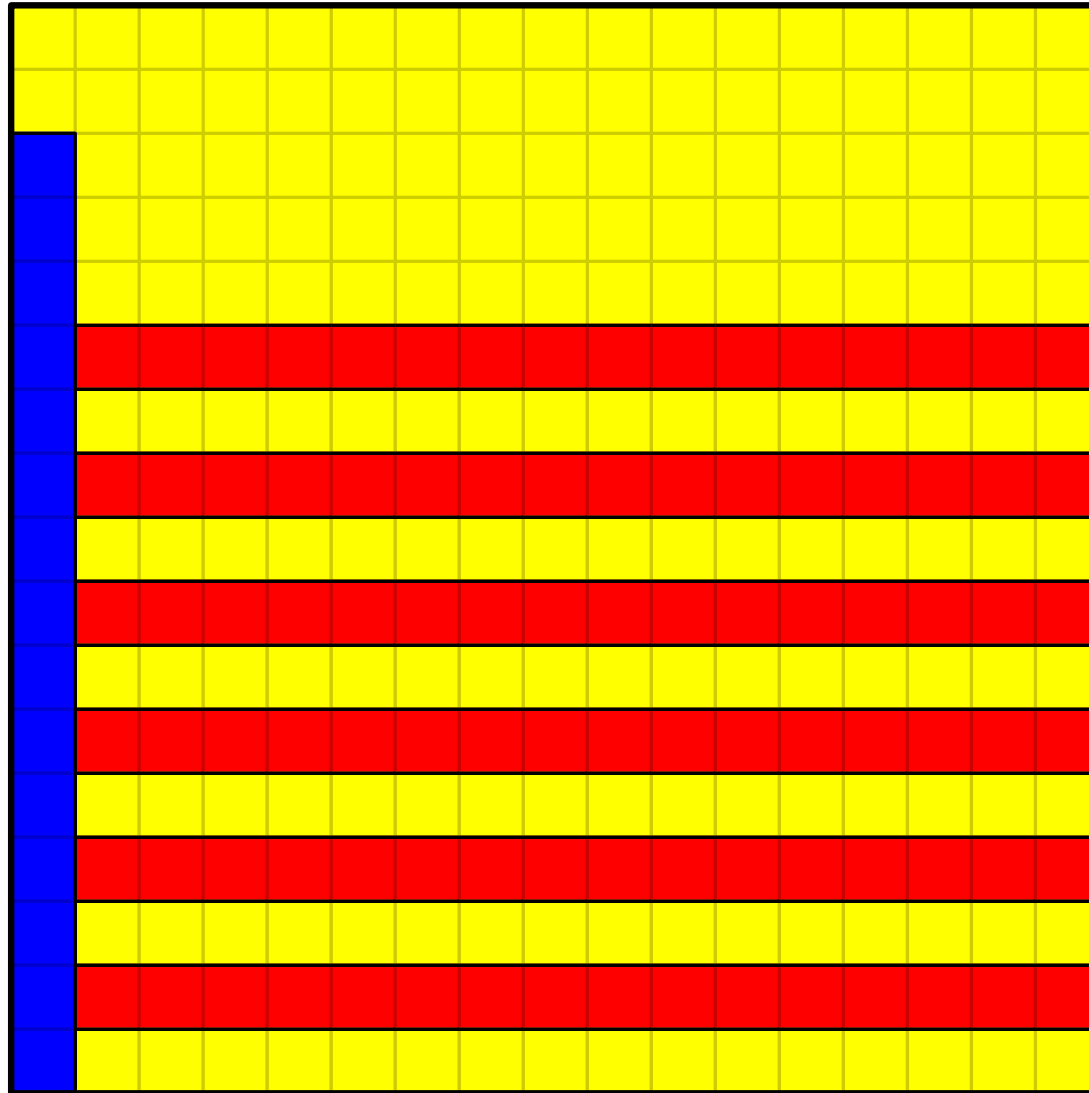
Greedy



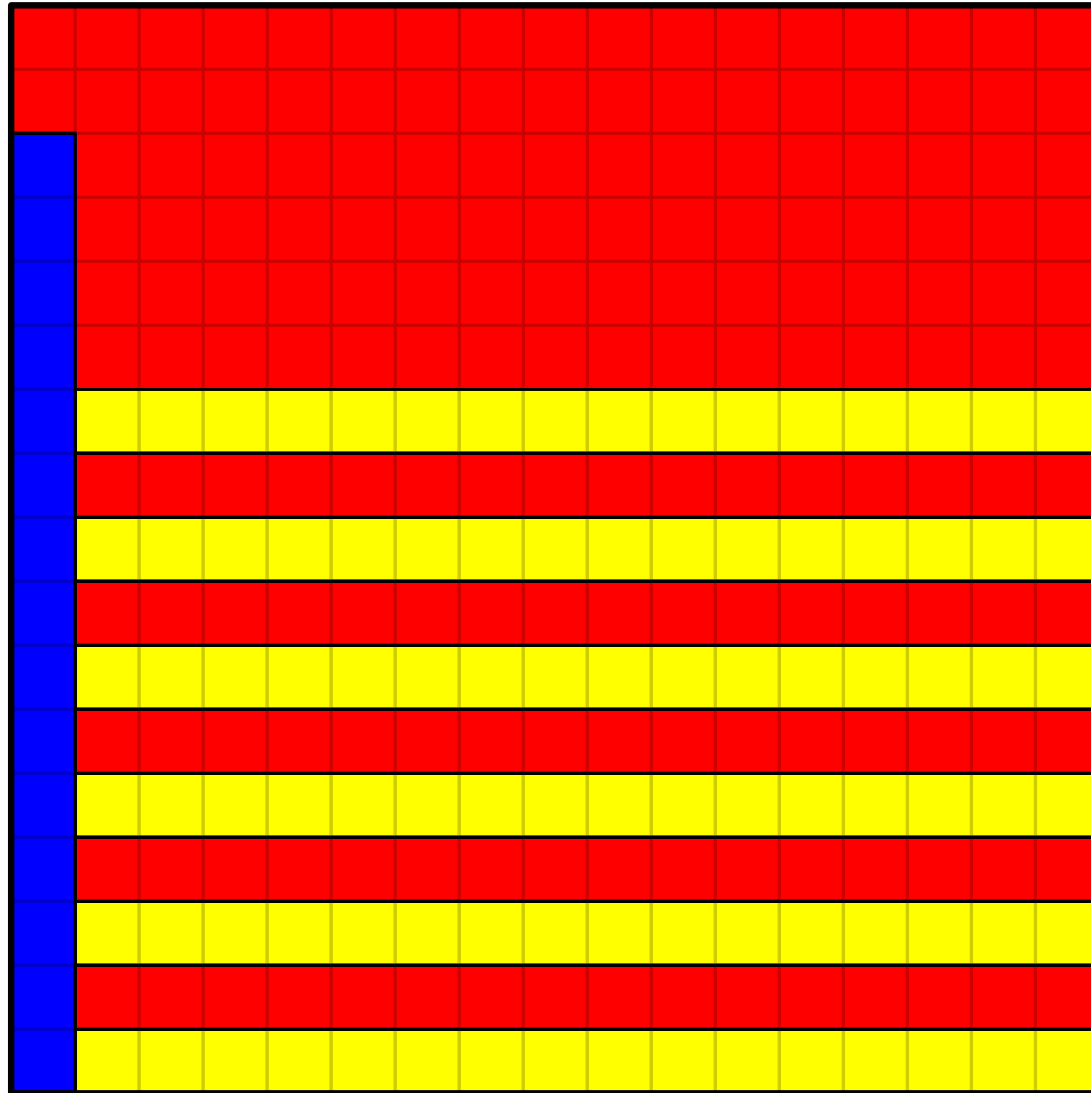
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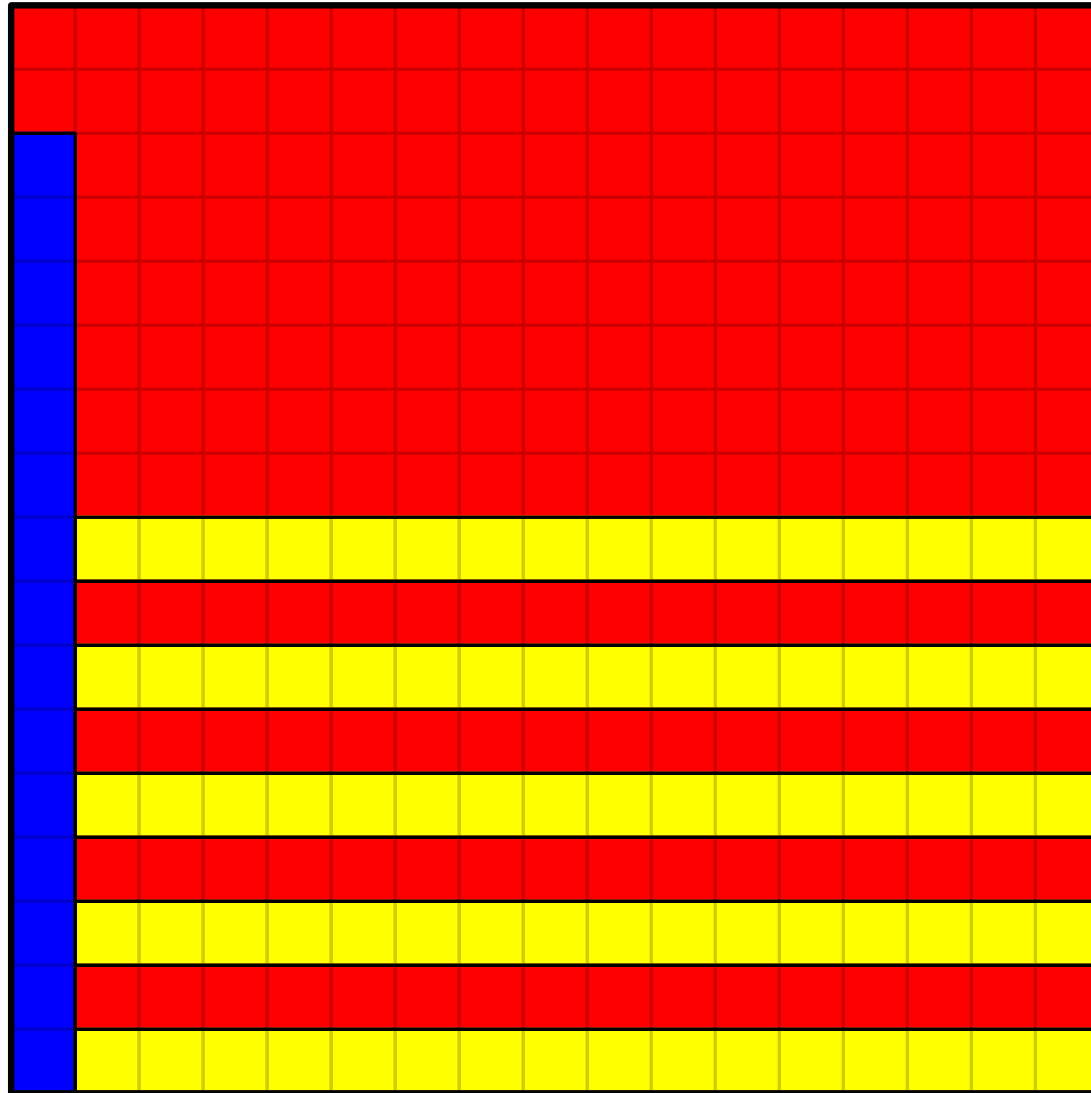
Greedy



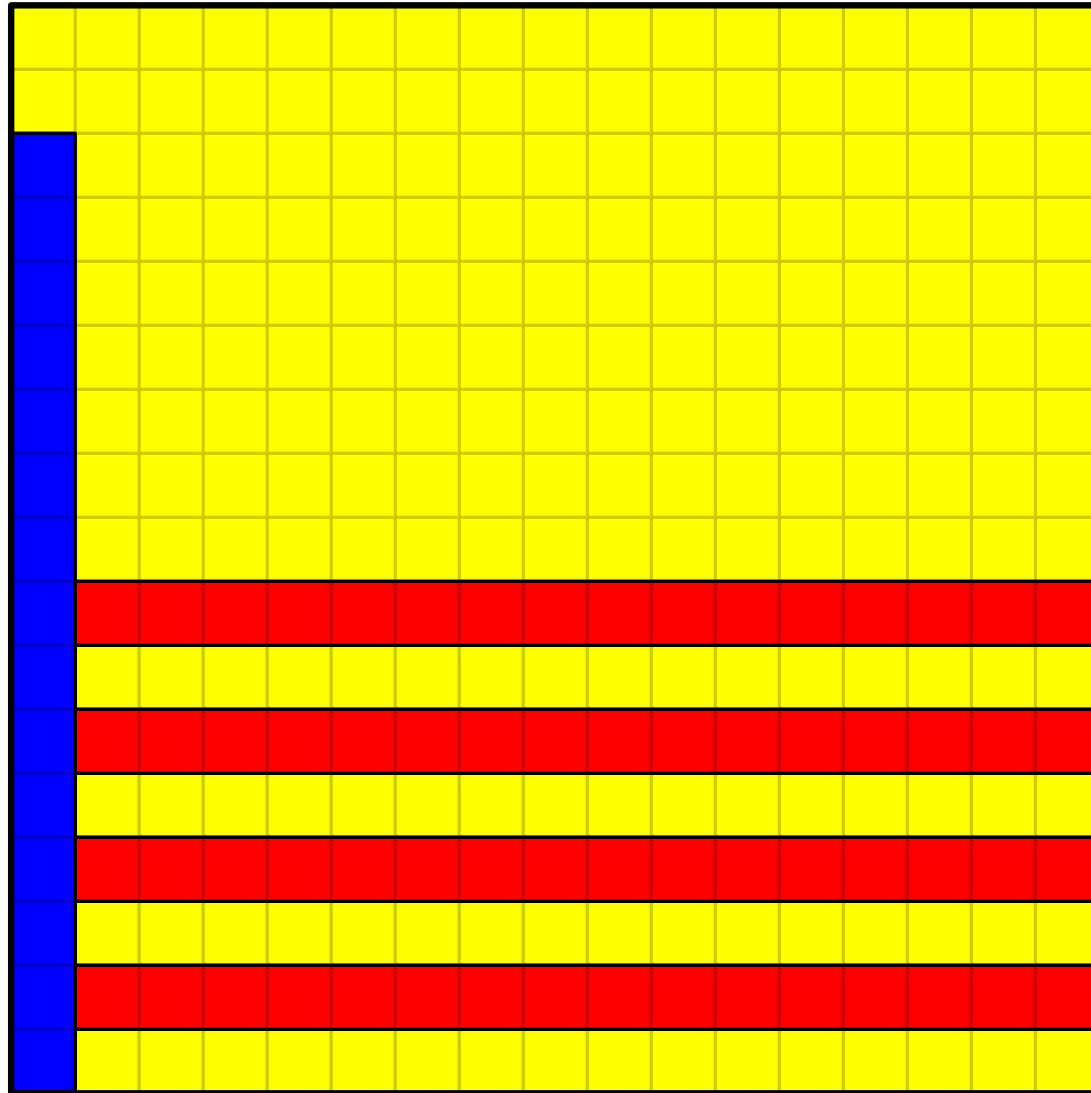
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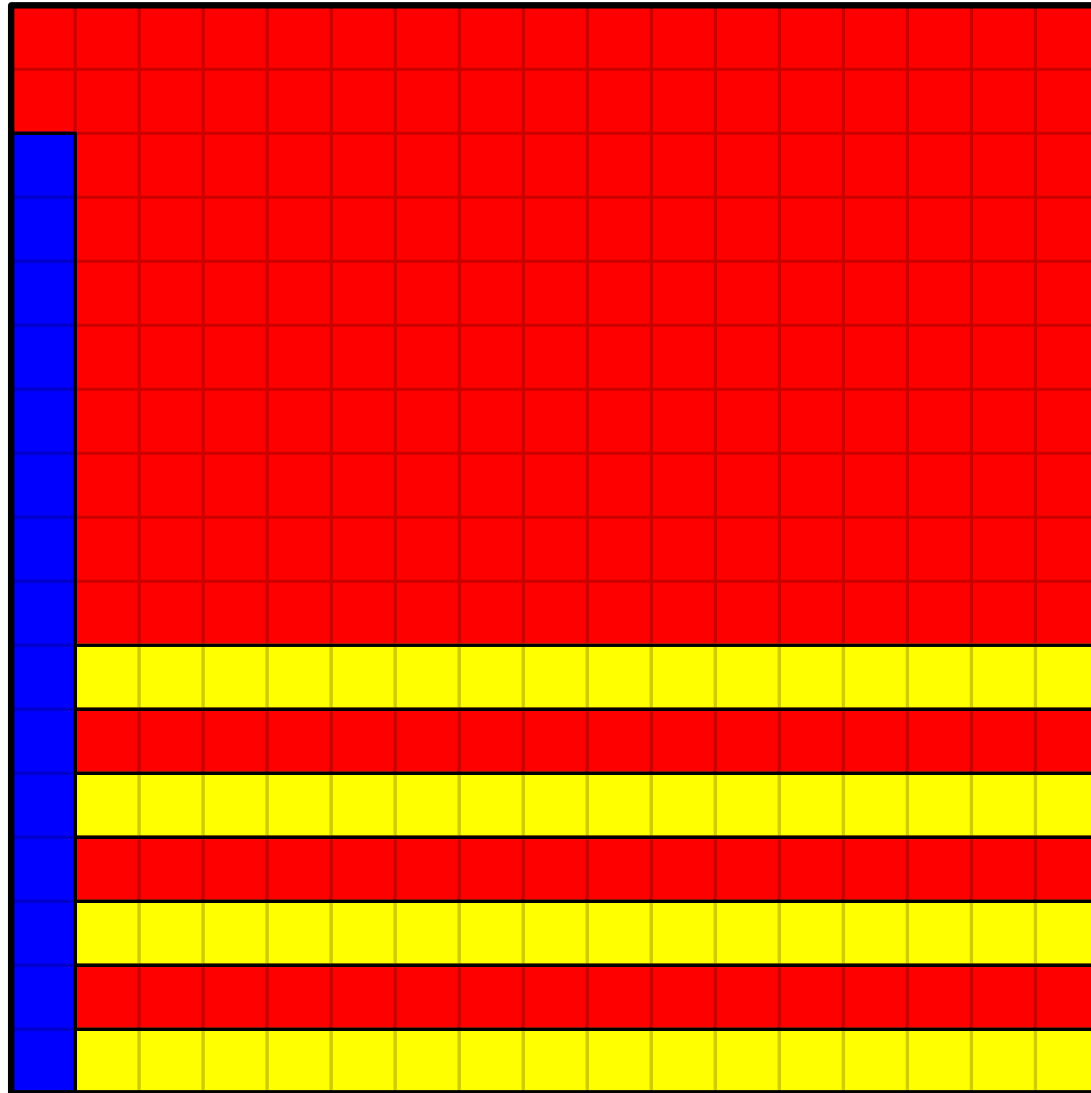
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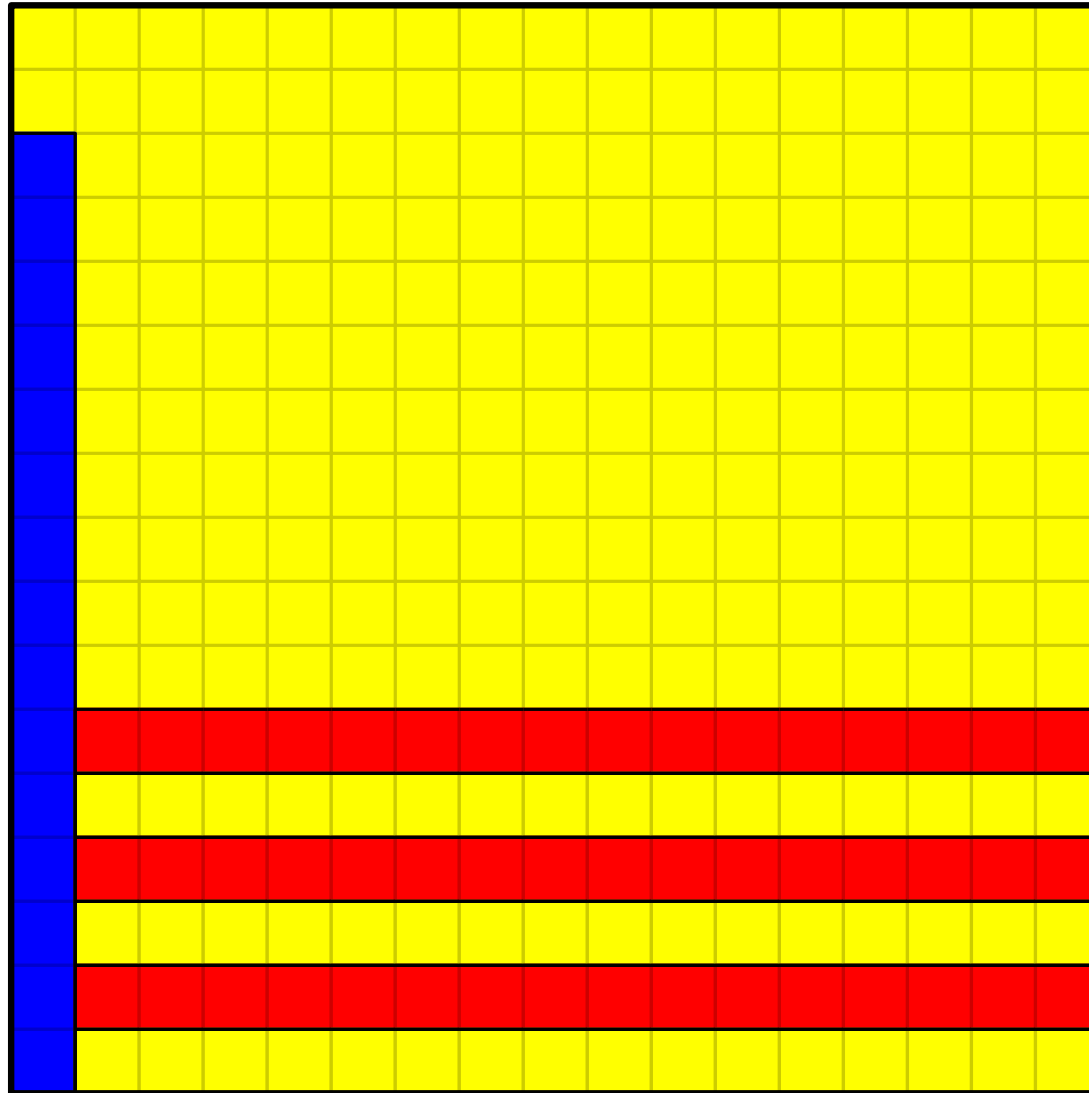
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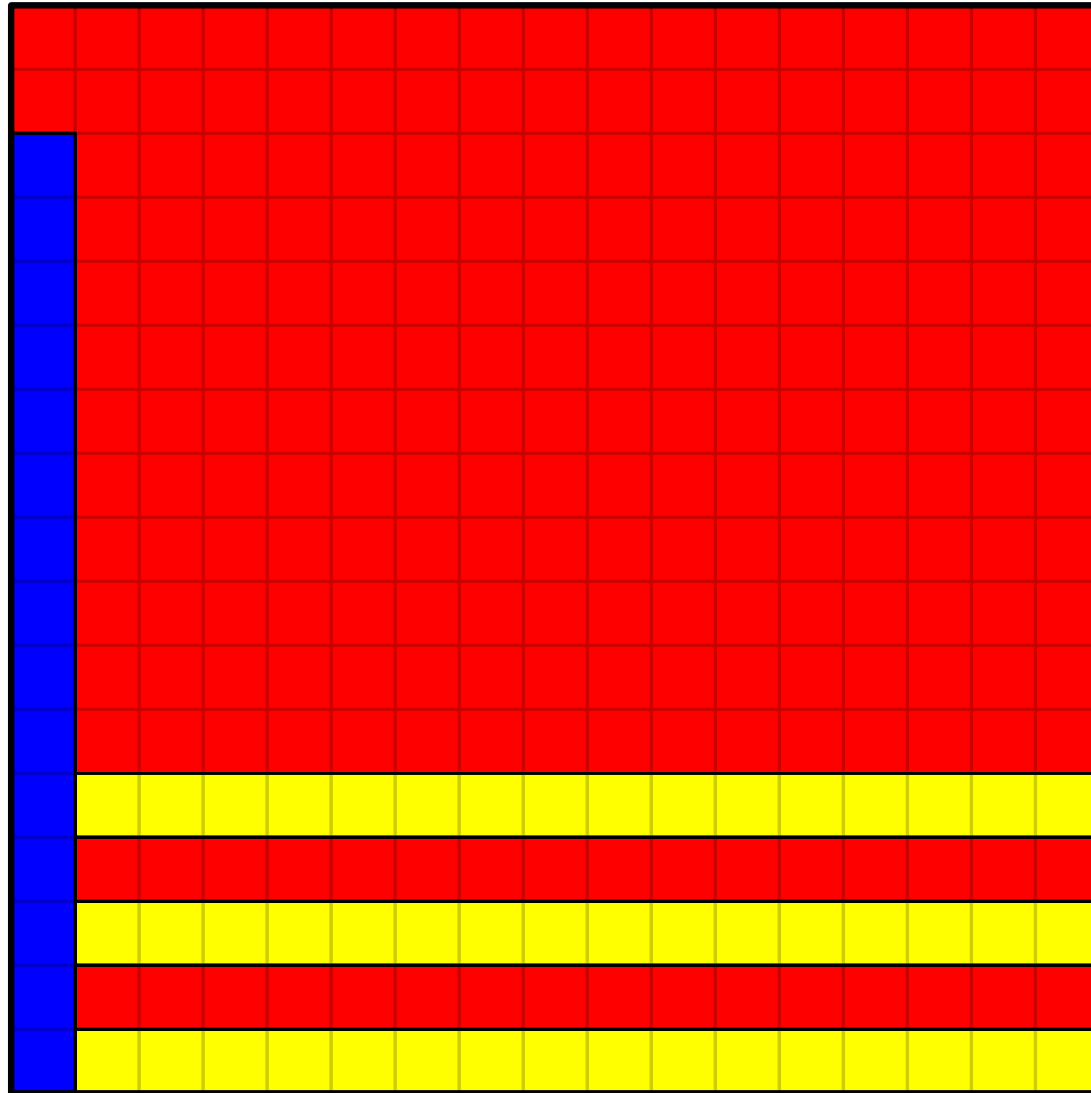
Greedy



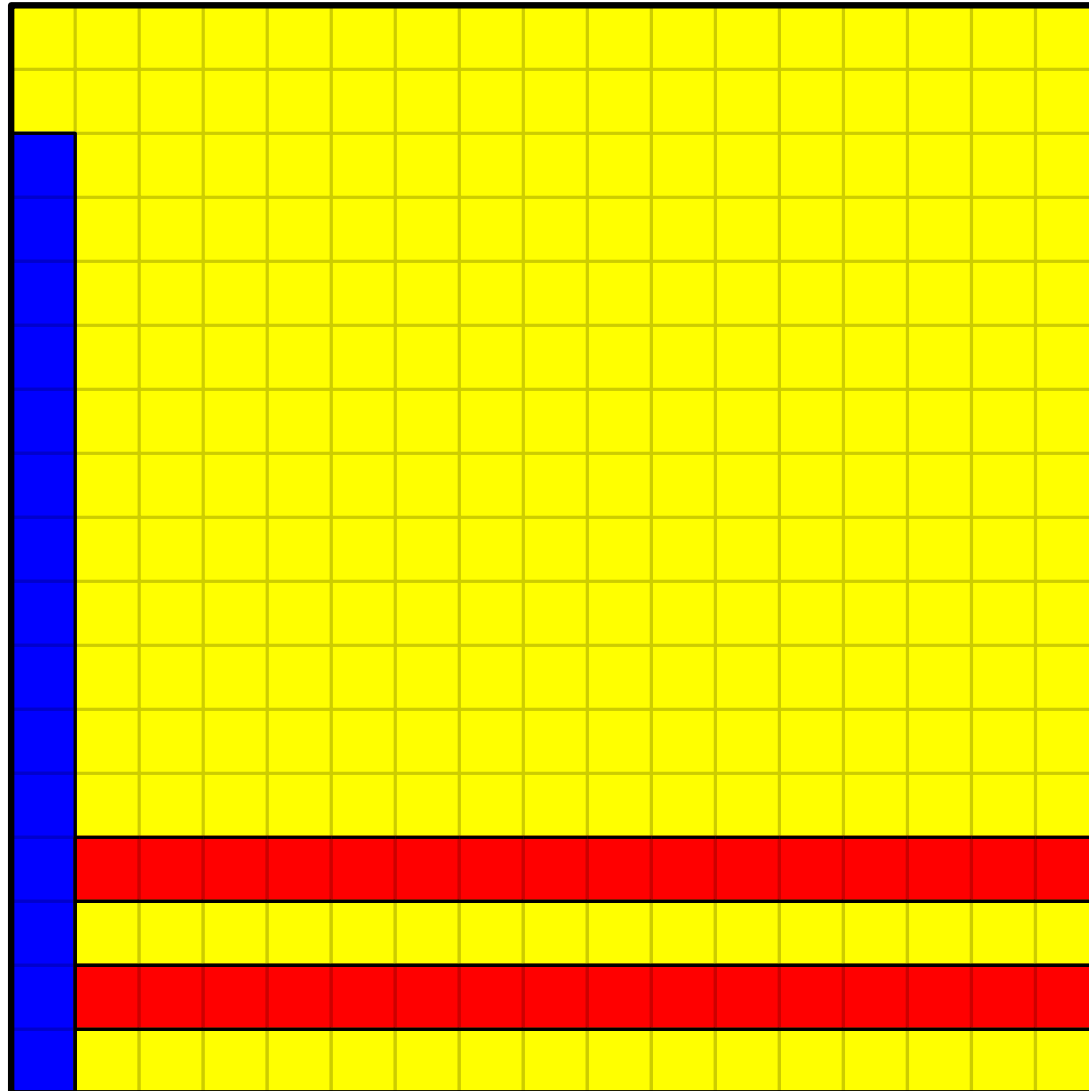
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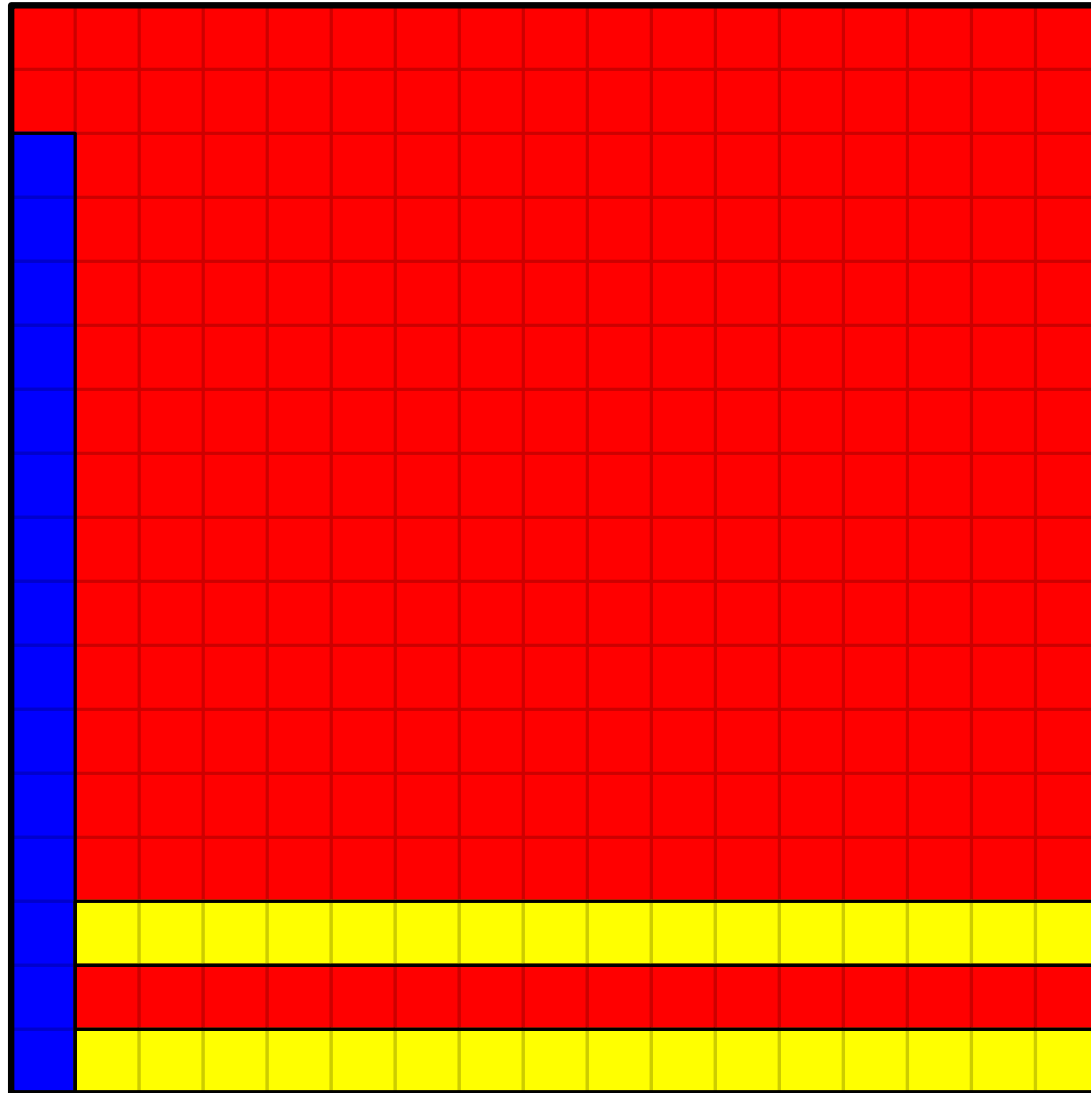
Greedy



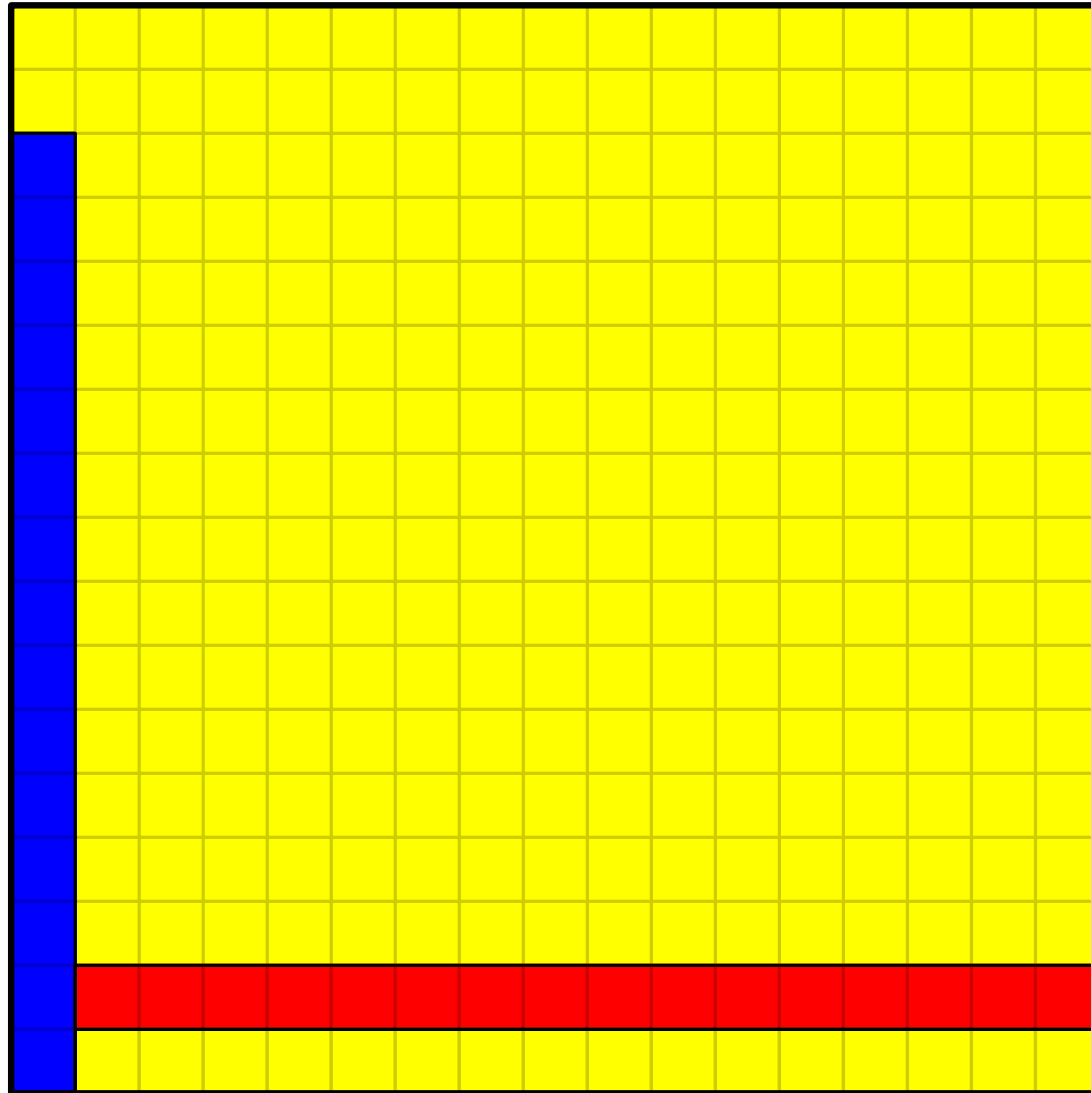
Greedy



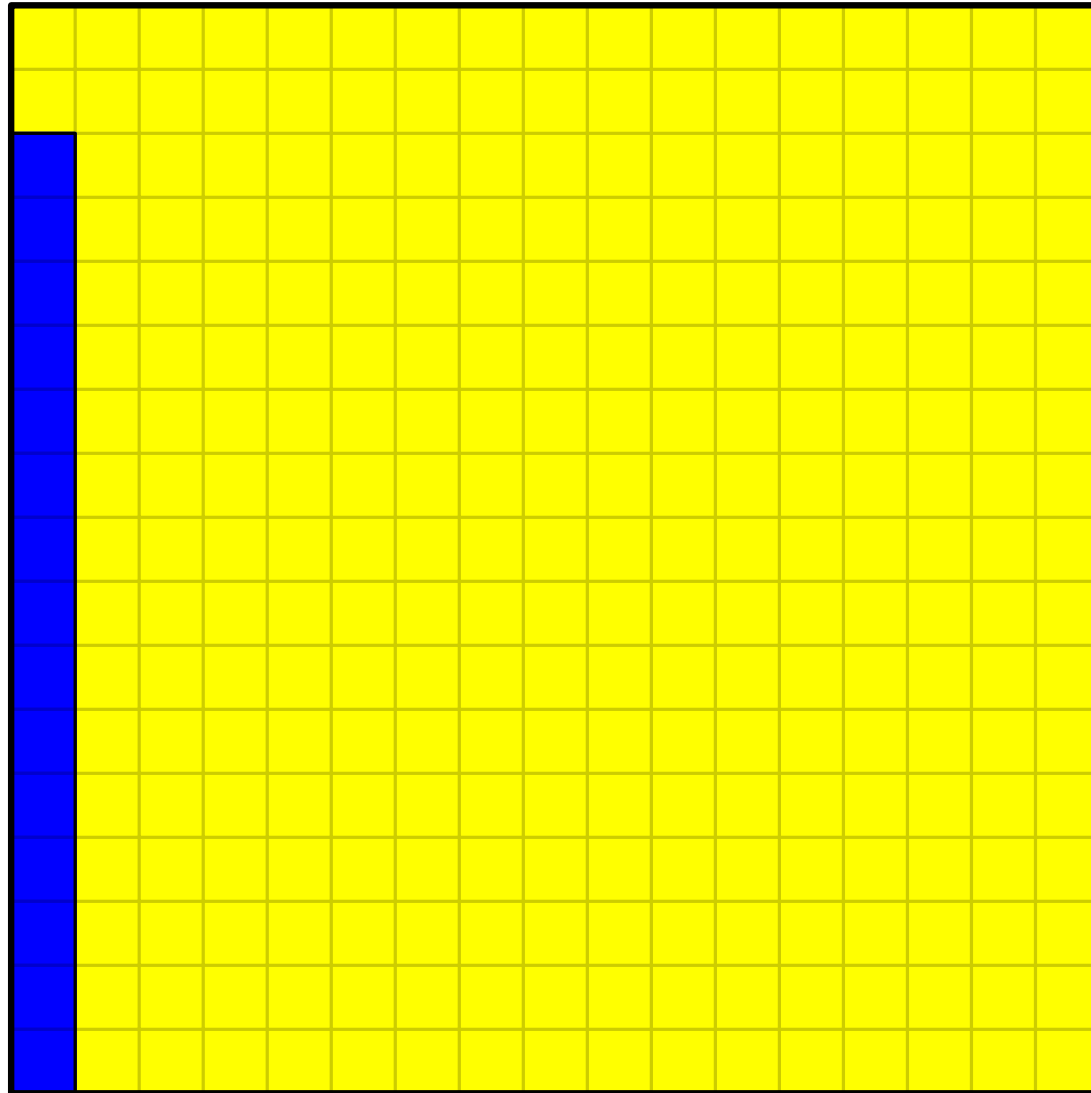
Greedy



Greedy



Greedy

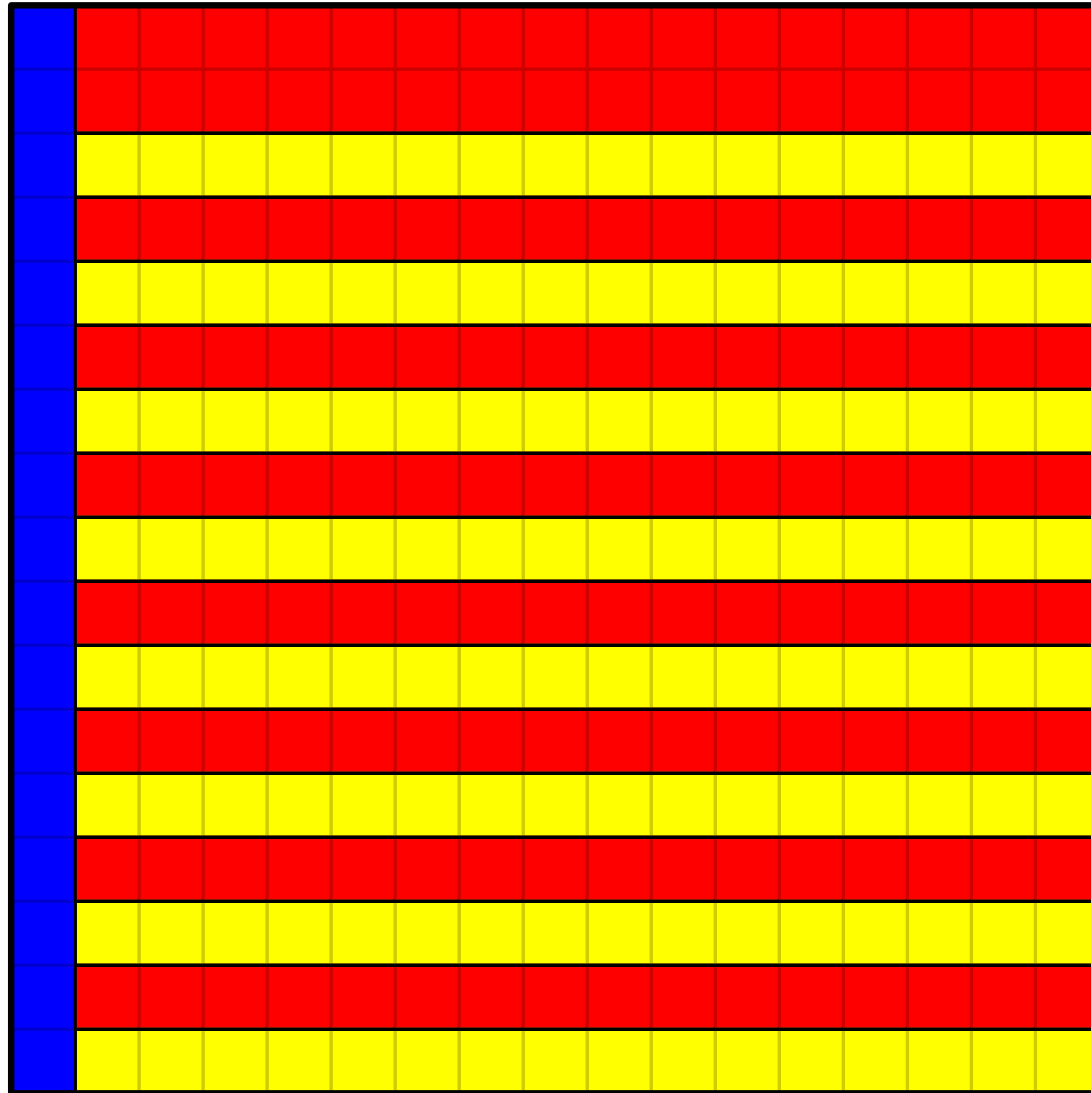


Greedy

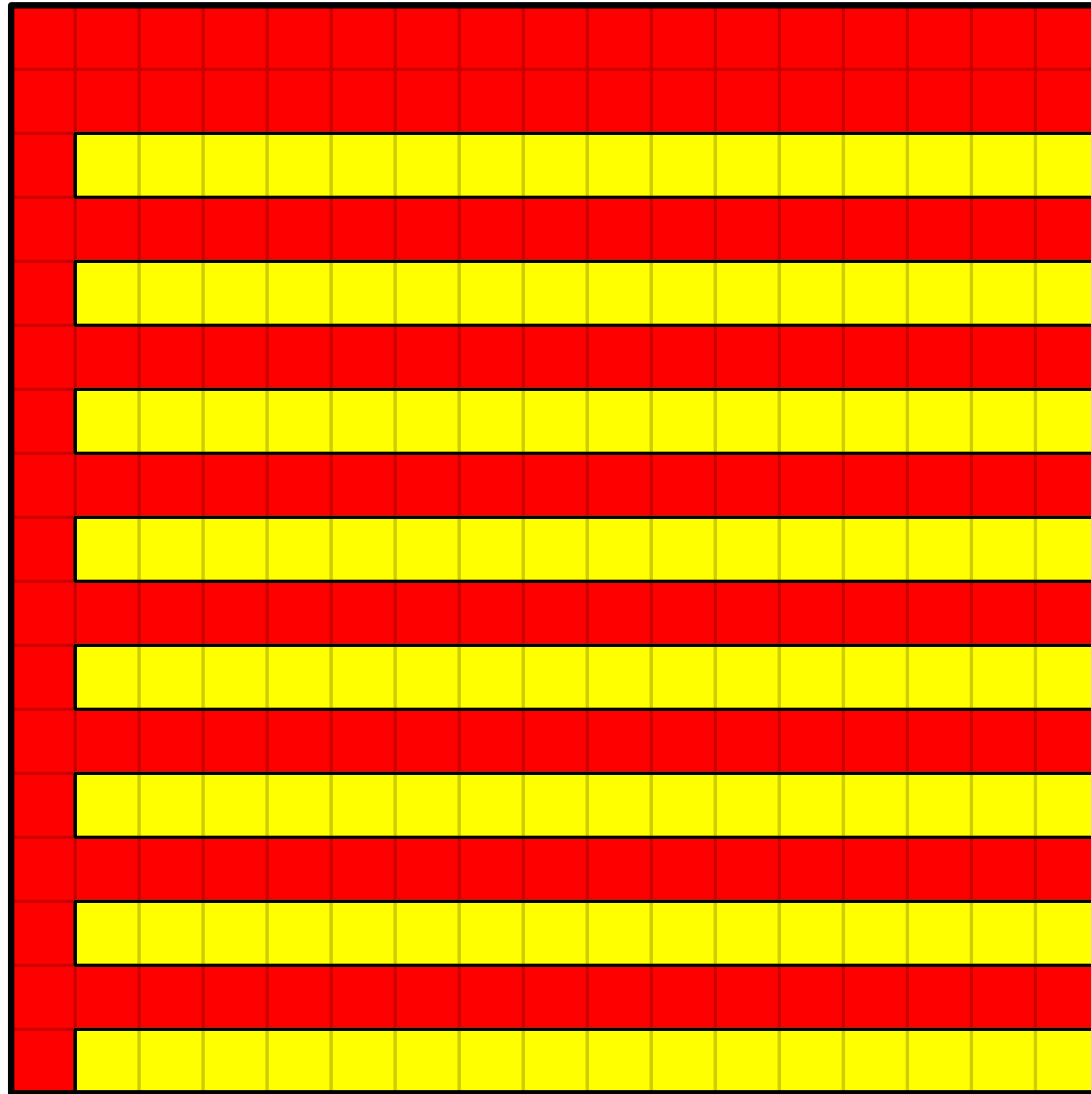
“Well done”

n moves

Greedy



Greedy

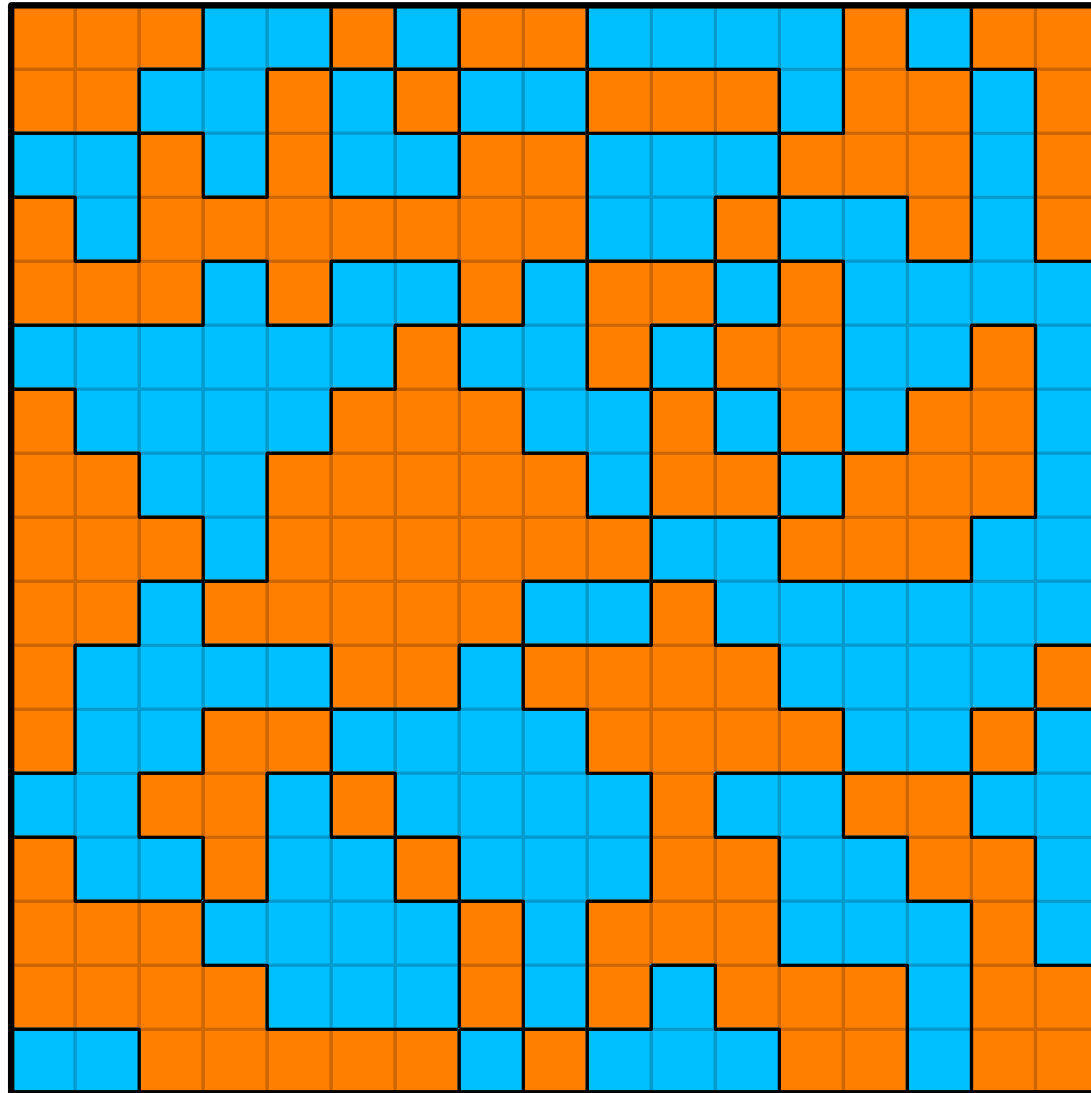


Greedy

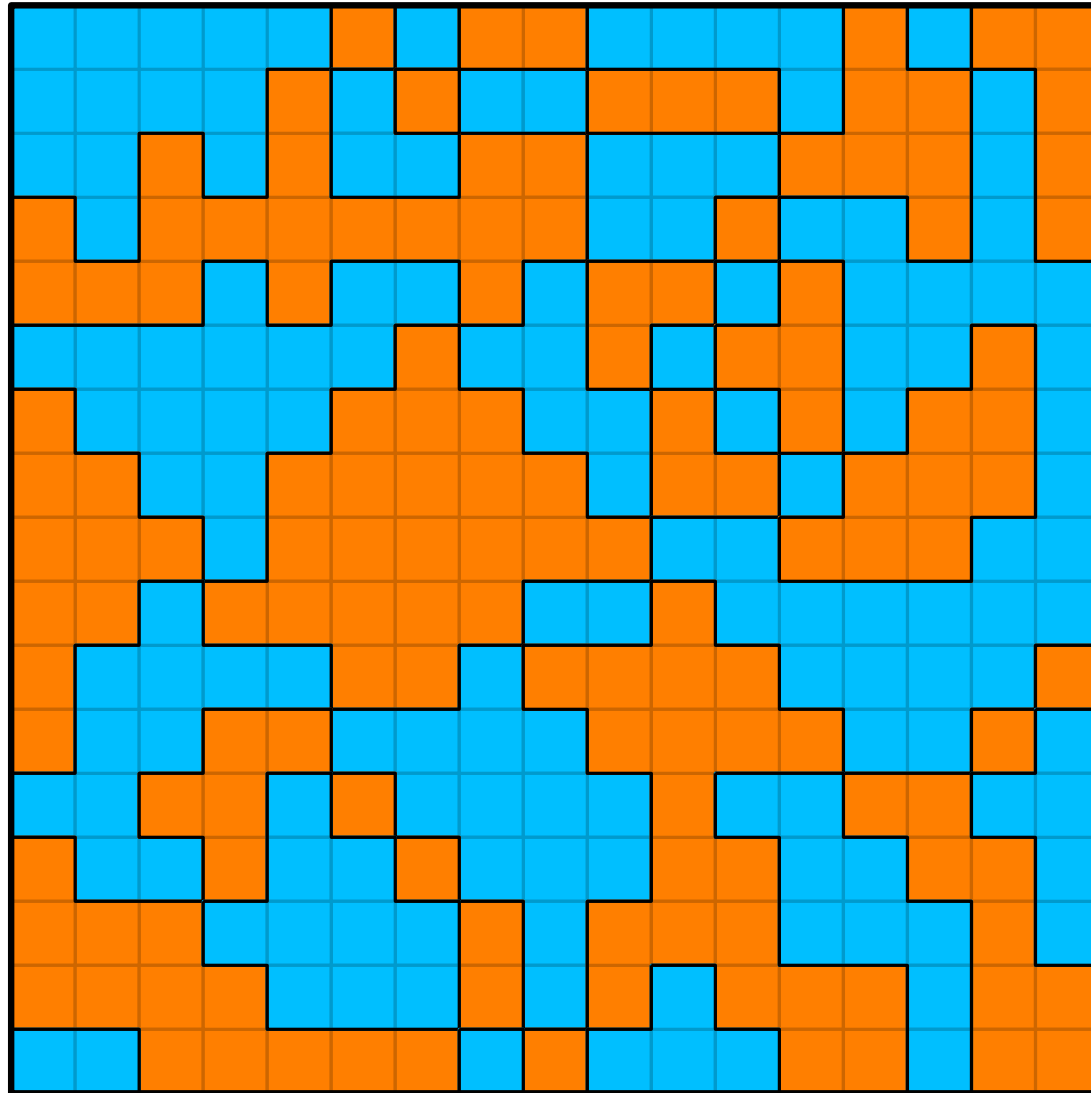
Much better!

3 moves

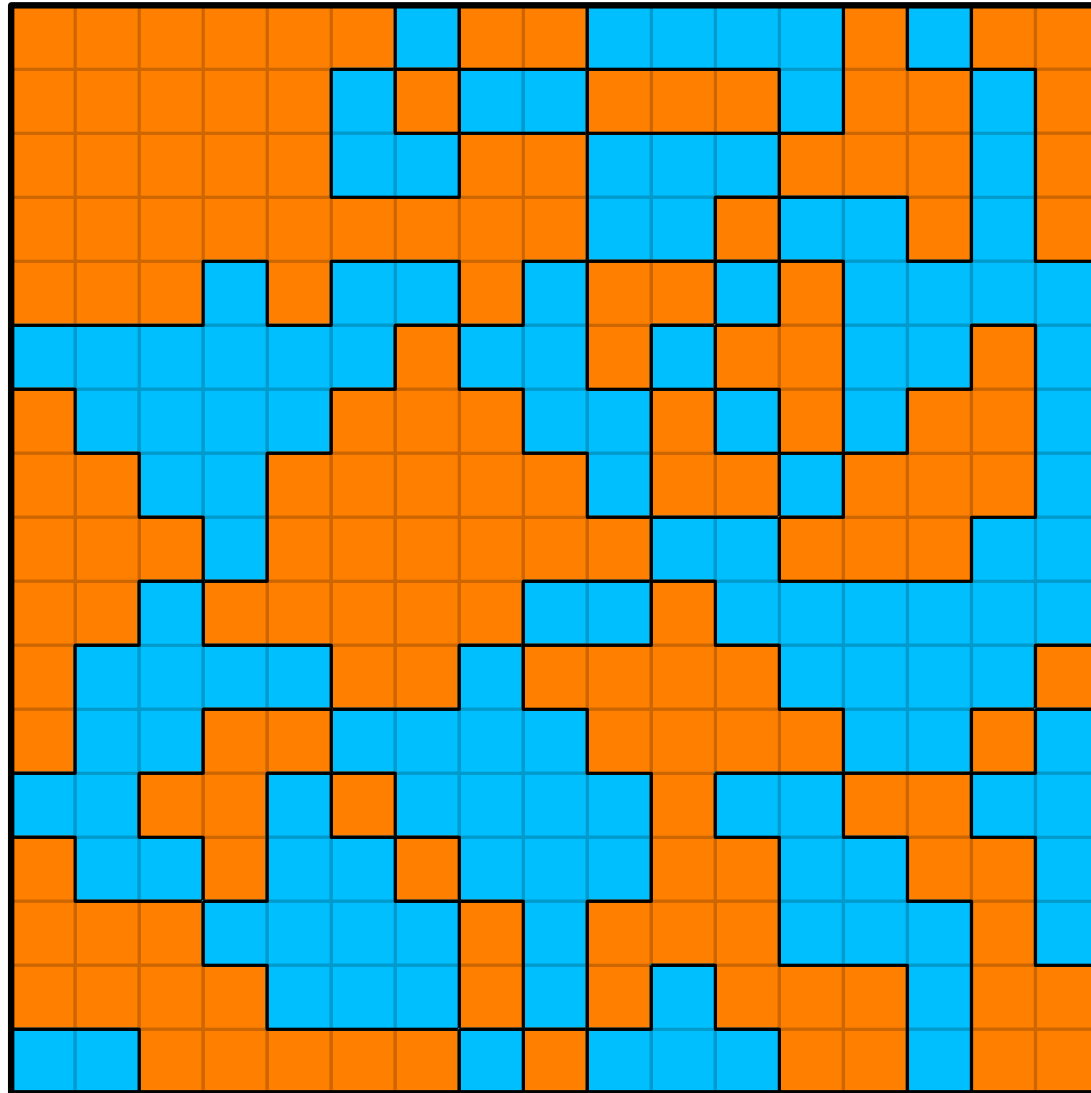
2 Colours



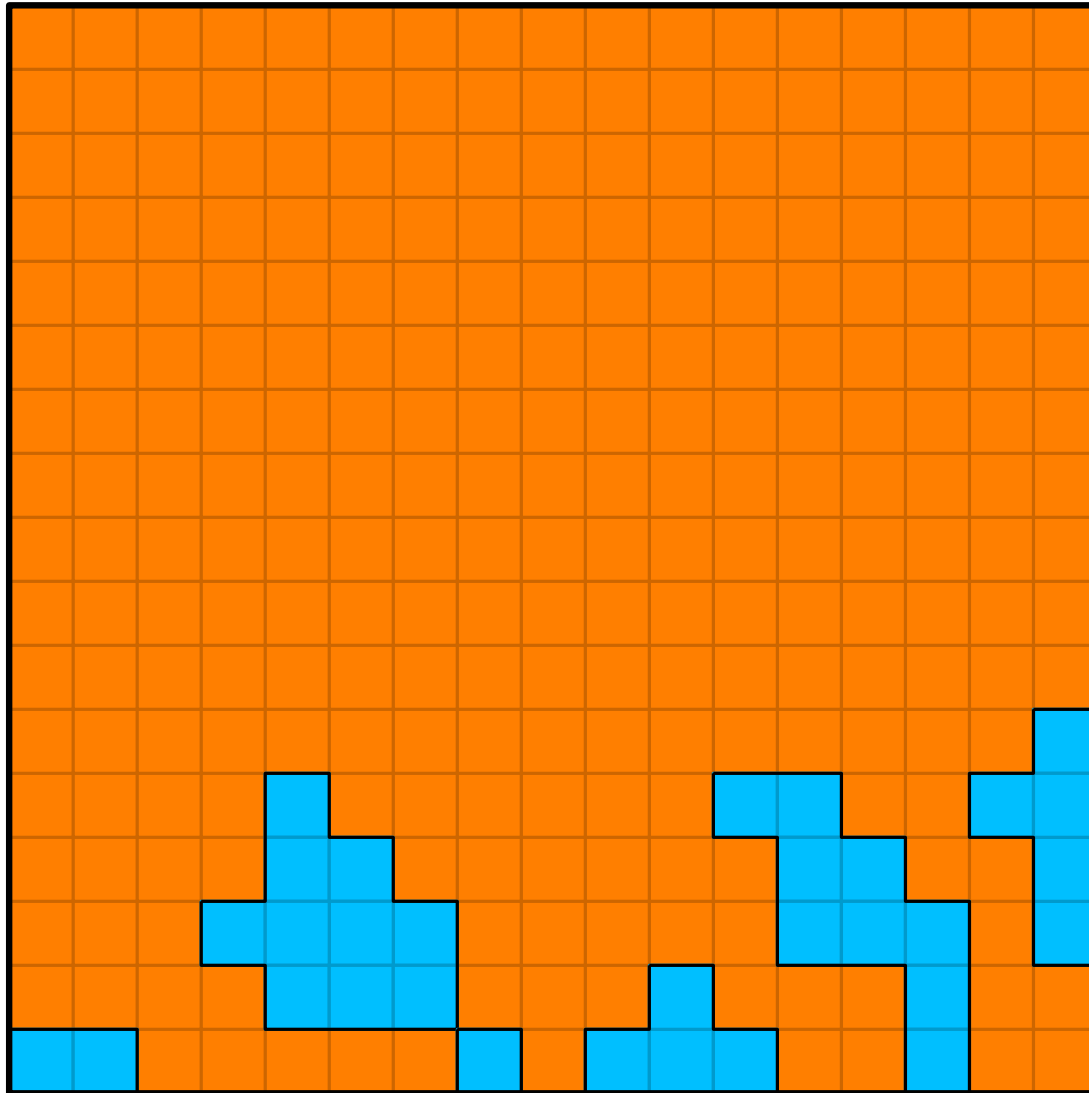
2 Colours



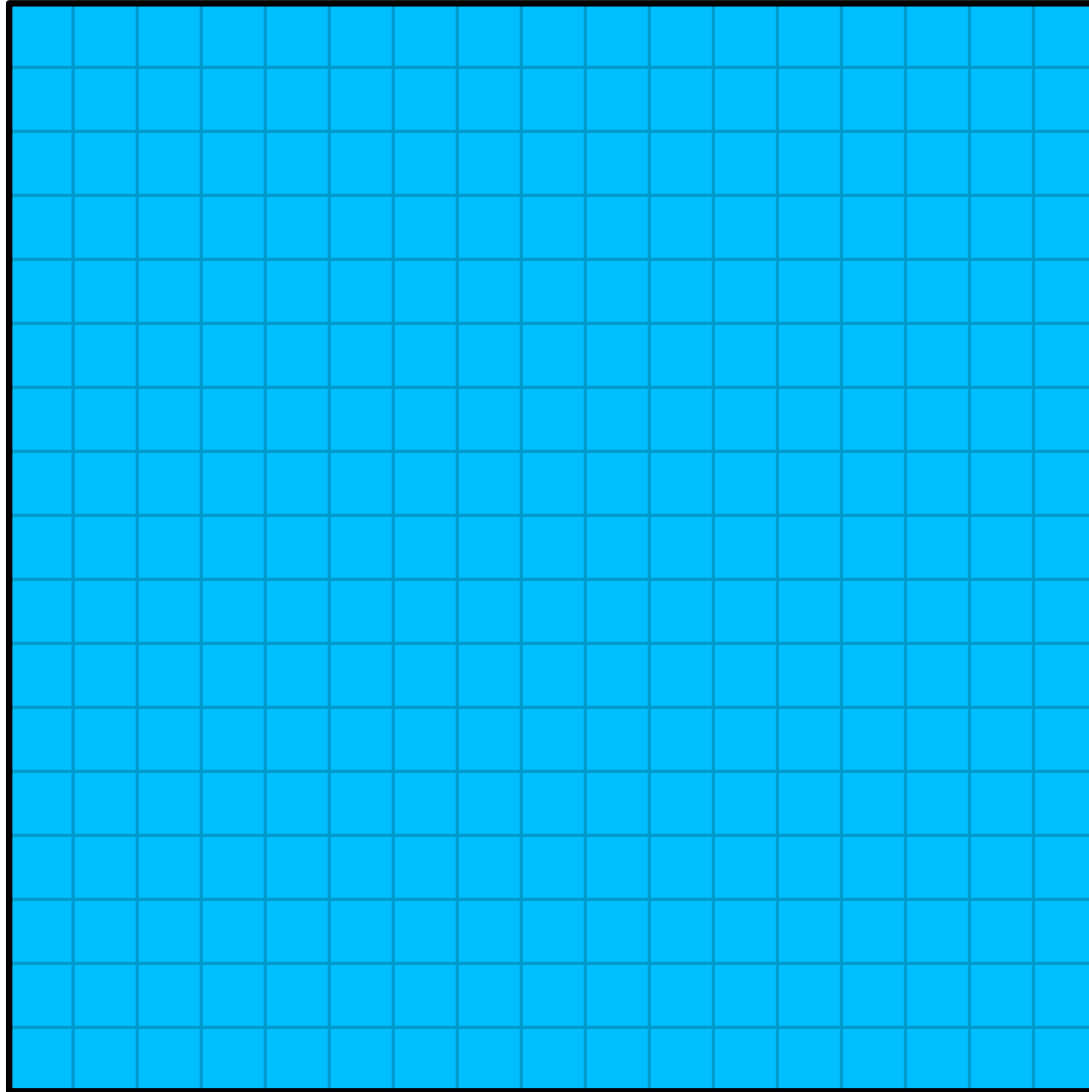
2 Colours



2 Colours



2 Colours



Shortest Common Supersequence

1. *a a b a c d c*

2. *c a b b a*

3. *b a d d d c a*

4. *a d d c a b*

5. *b a d c c d a a*

6. *d c a a b d c*

7. *c a b a d*

8. *b c d a a b c*

Shortest Common Supersequence

1. *a a b a c d c*

2. *c a b b a*

3. *b a d d d c a*

4. *a d d c a b*

5. *b a d c c d a a*

6. *d c a a b d c*

7. *c a b a d*

8. *b c d a a b c*

a a b c a d c a d a c b c a b d c a a

Shortest Common Supersequence

1. *a a b a c d c*

2. *c a b b a*

3. *b a d d d c a*

4. *a d d c a b*

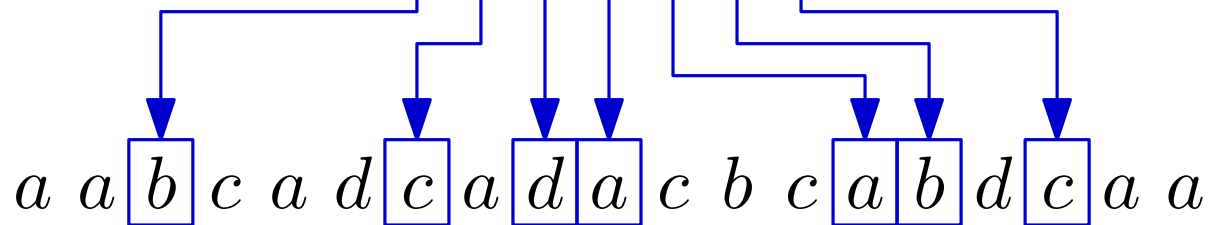
5. *b a d c c d a a*

6. *d c a a b d c*

7. *c a b a d*

8.

<i>b</i>	<i>c</i>	<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>
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Shortest Common Supersequence

1. *a a b a c d c*

2. *c a b b a*

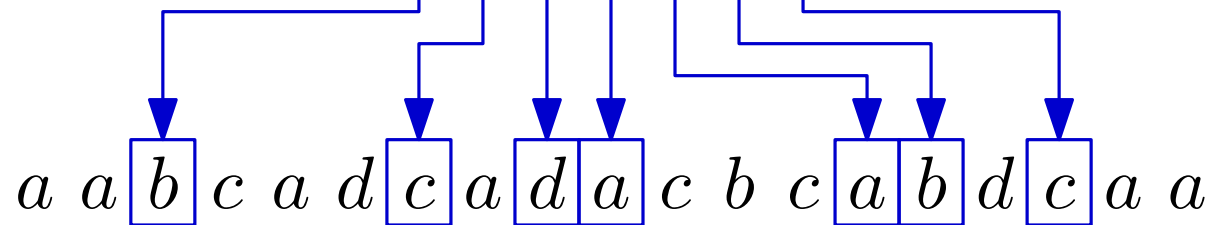
- **NP**-hard, even with a binary alphabet,
- no polynomial-time constant factor approximation algorithm, unless **P = NP**.

6. *d c a a b d c*

7. *c a b a d*

8.

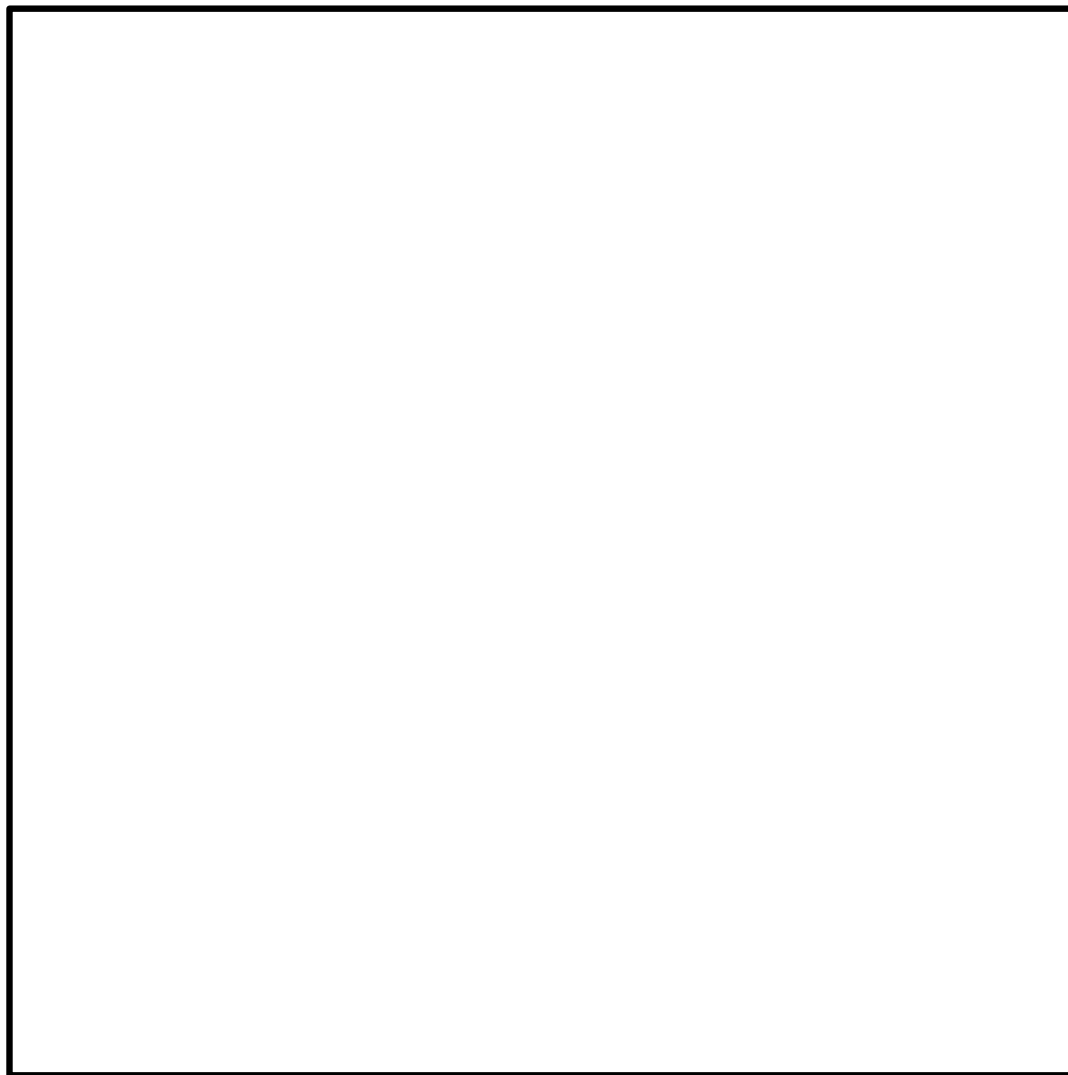
<i>b</i>	<i>c</i>	<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>
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4 or More Colours

a *b* *b* *a*

a *b*



4 or More Colours

a

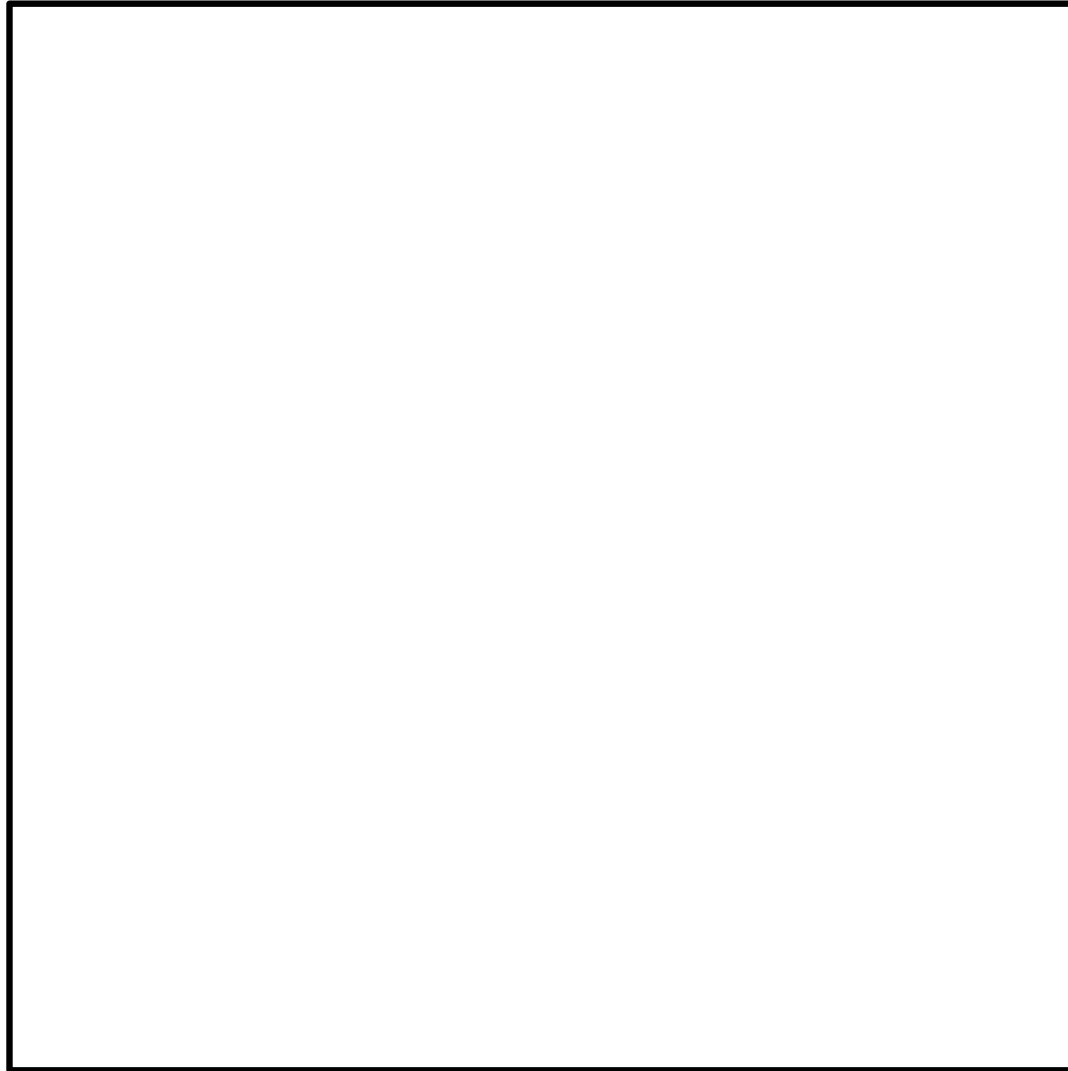
b

b

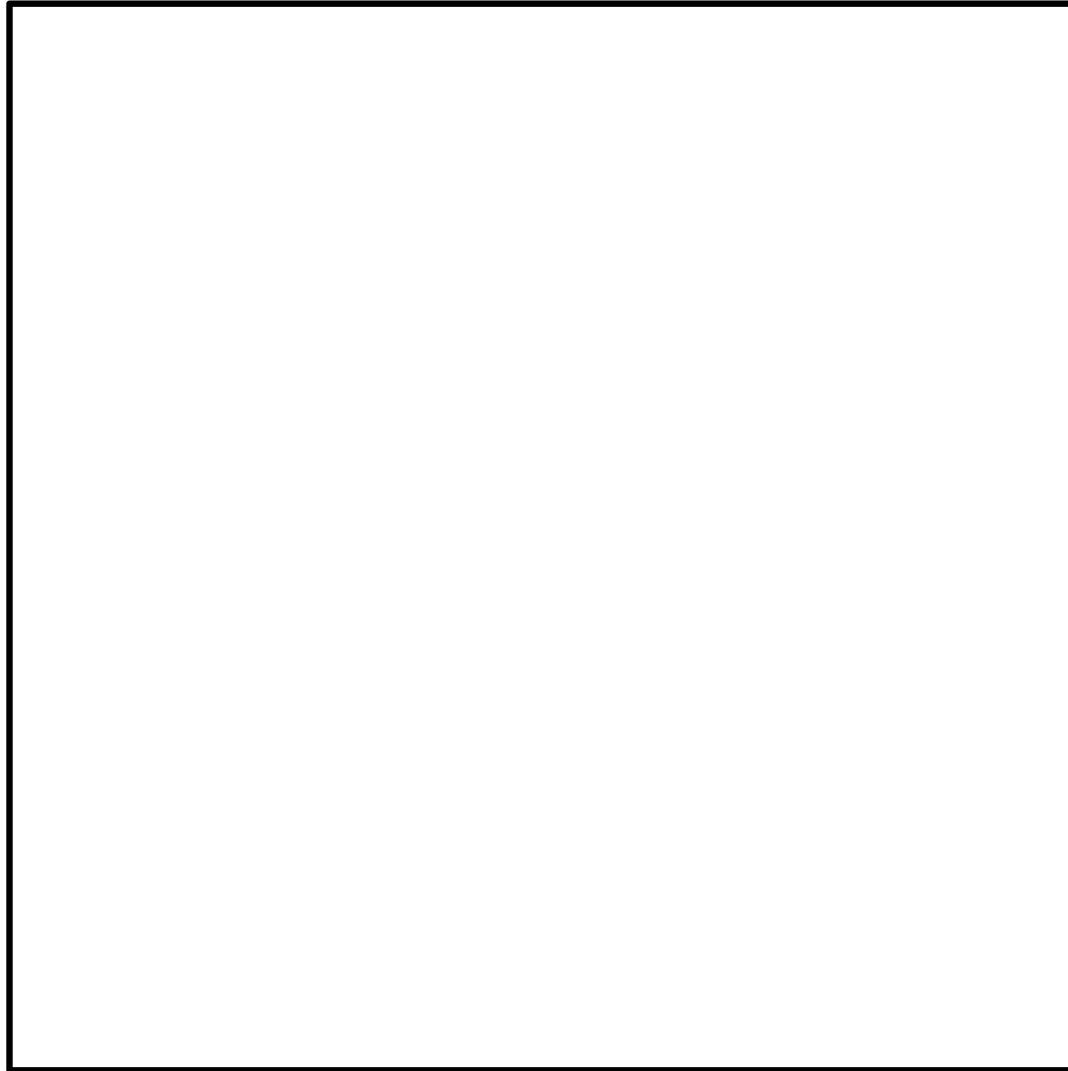
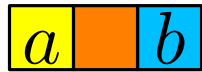
a

a

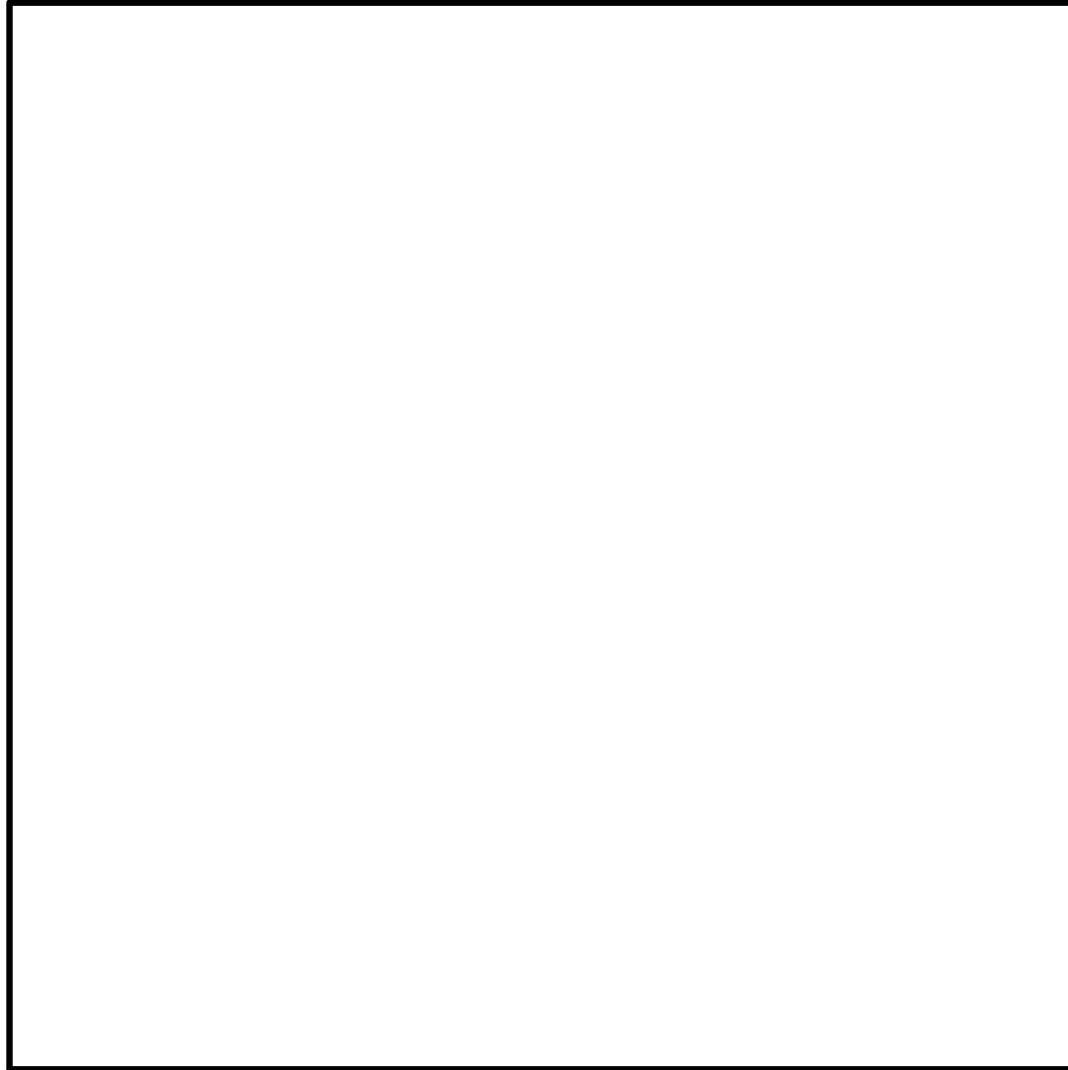
b



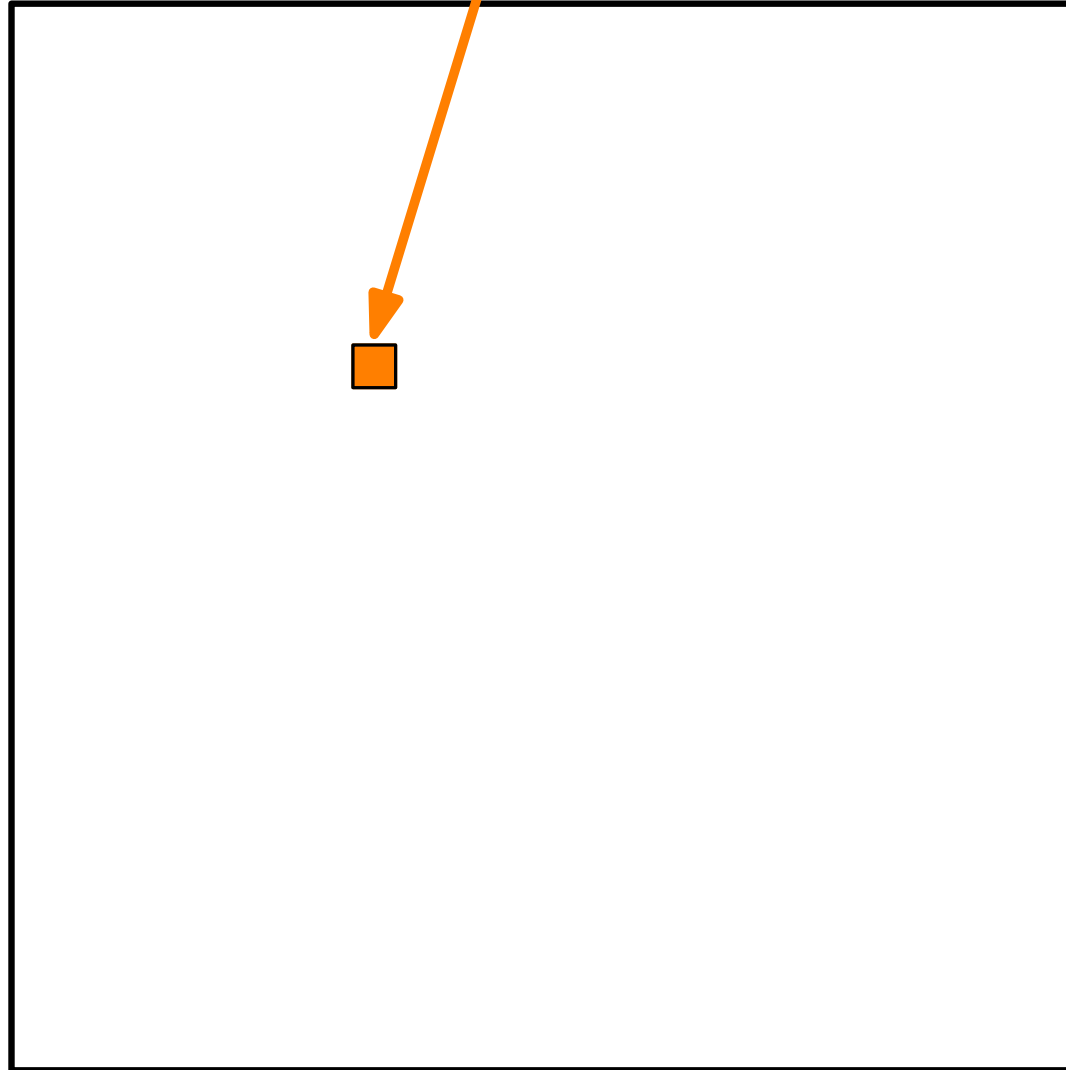
4 or More Colours



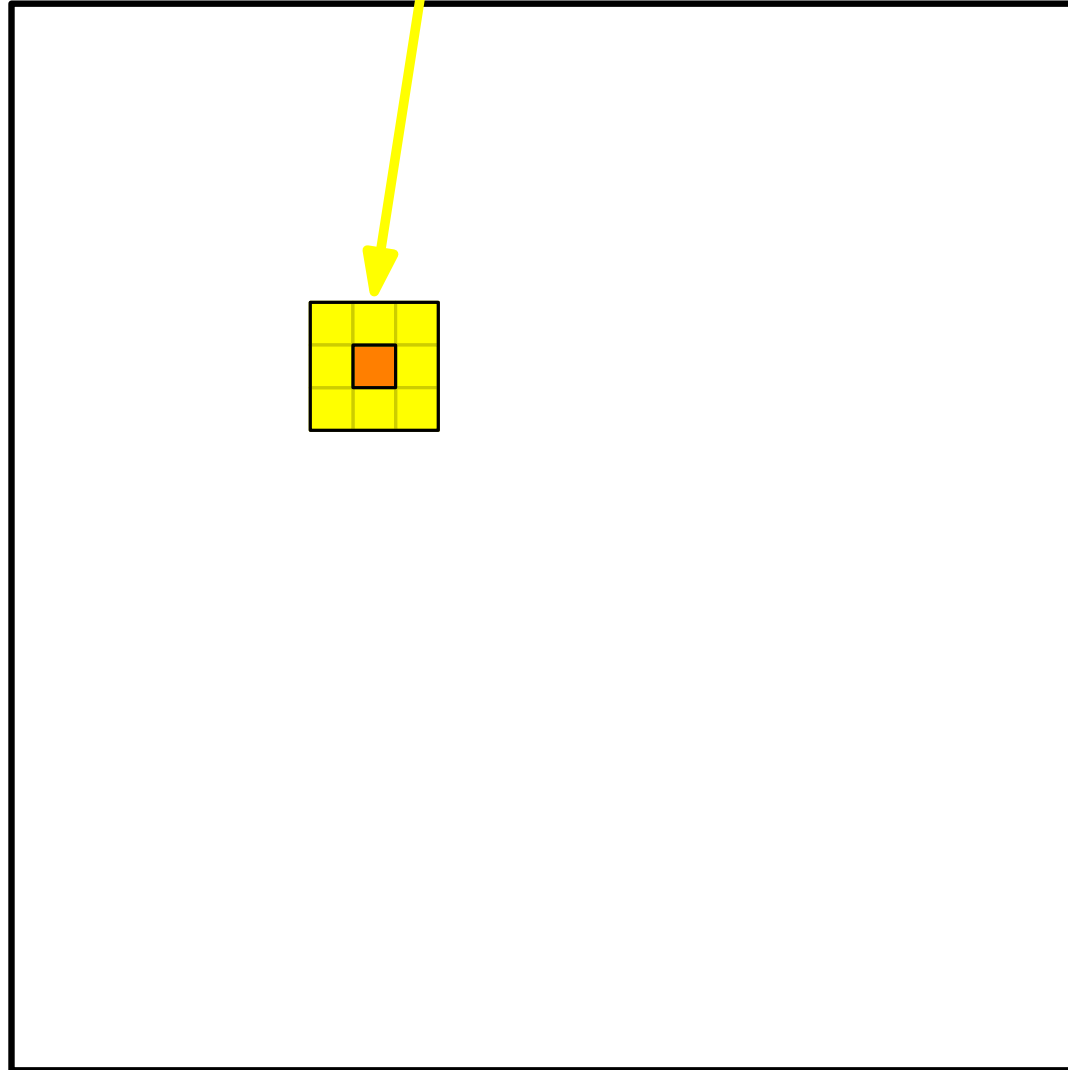
4 or More Colours



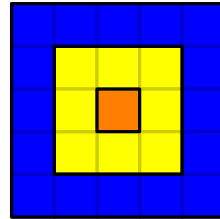
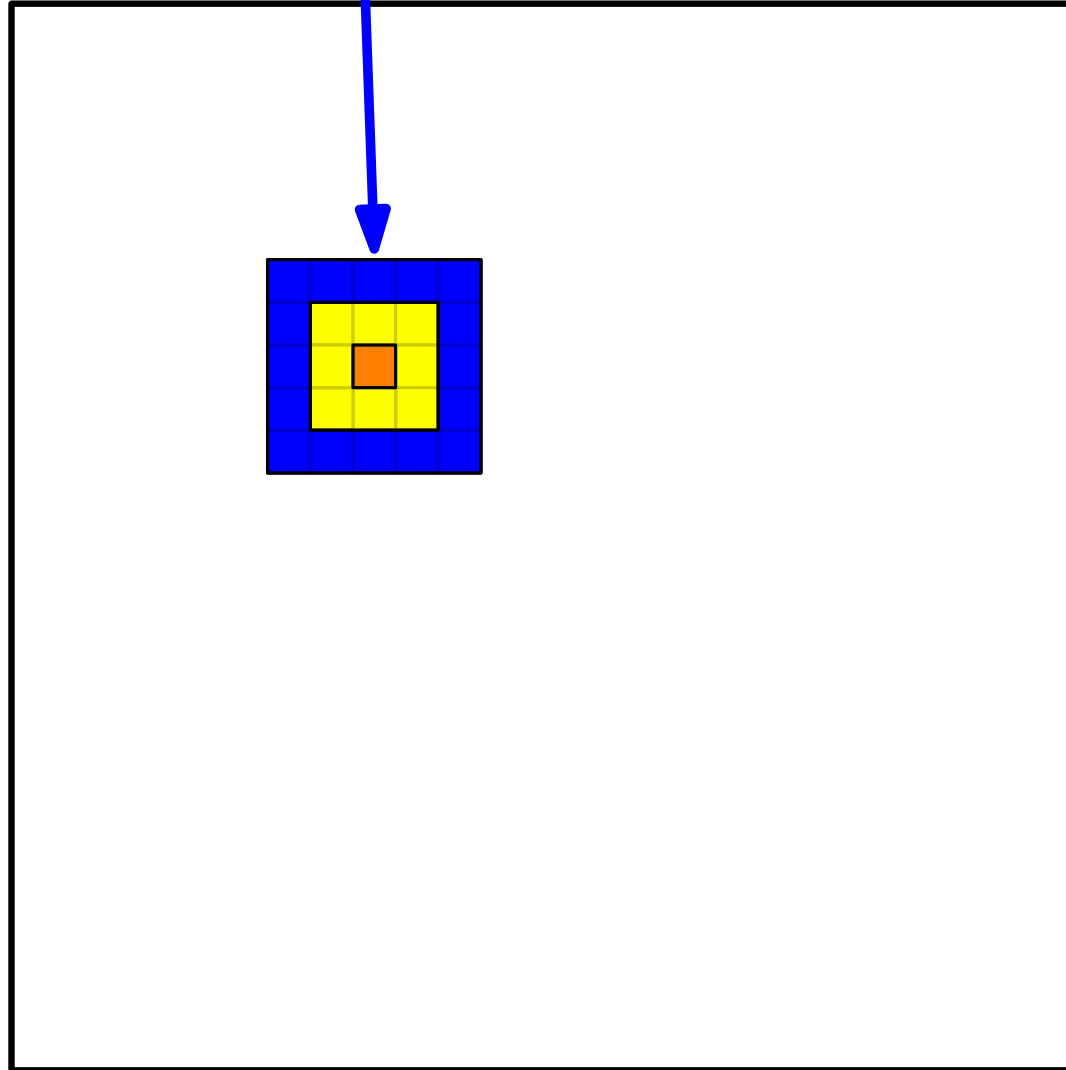
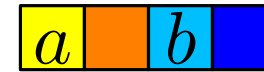
4 or More Colours



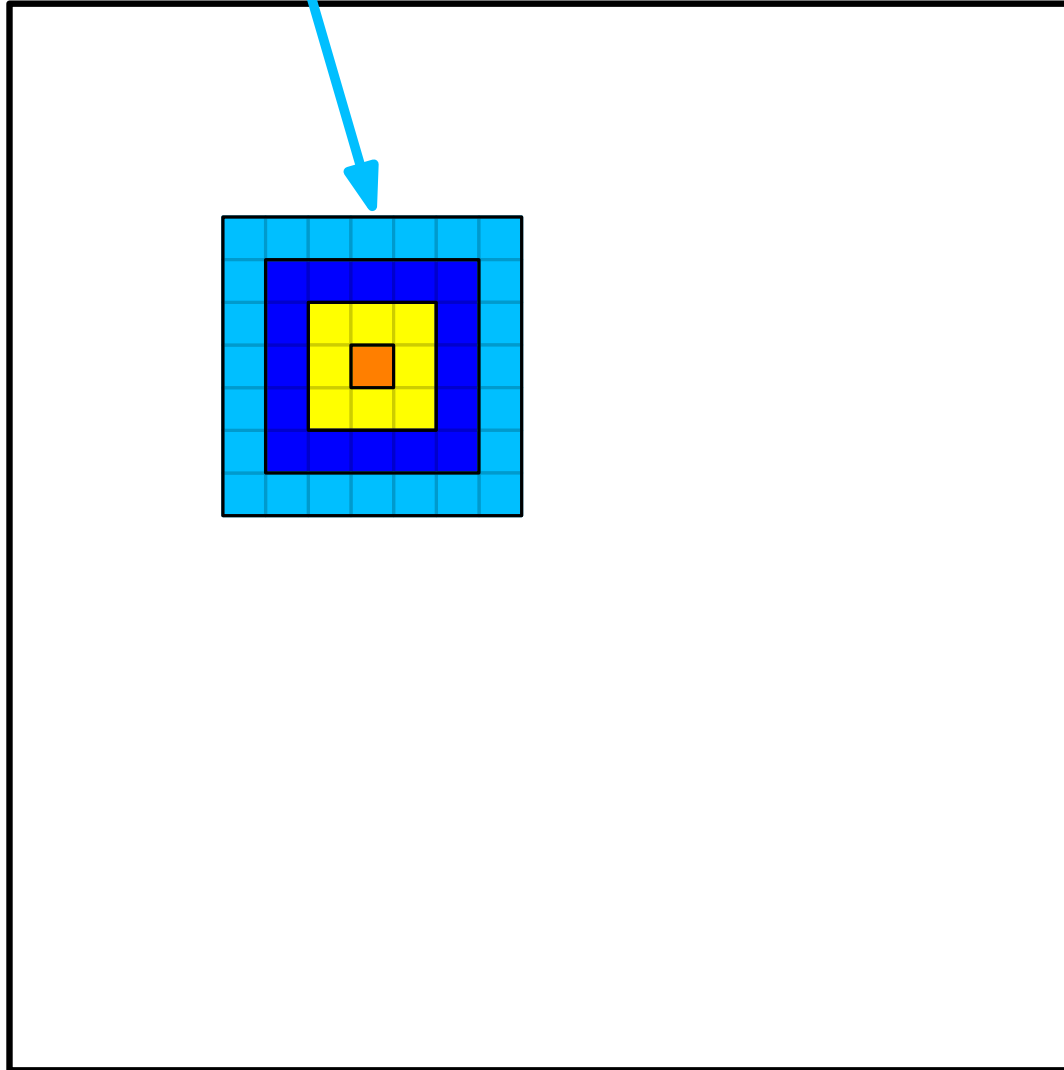
4 or More Colours



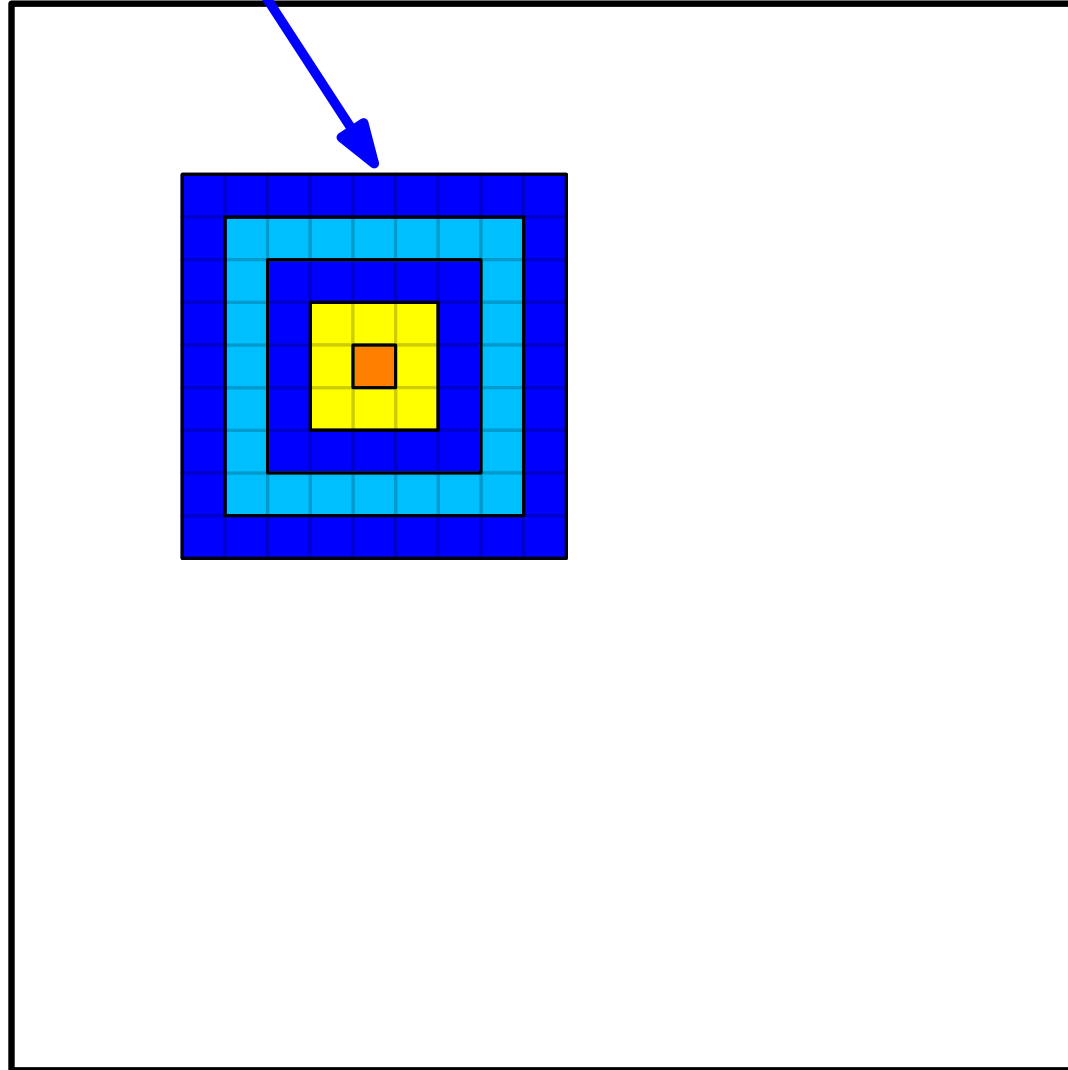
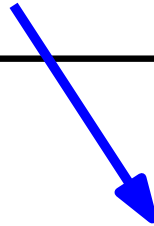
4 or More Colours



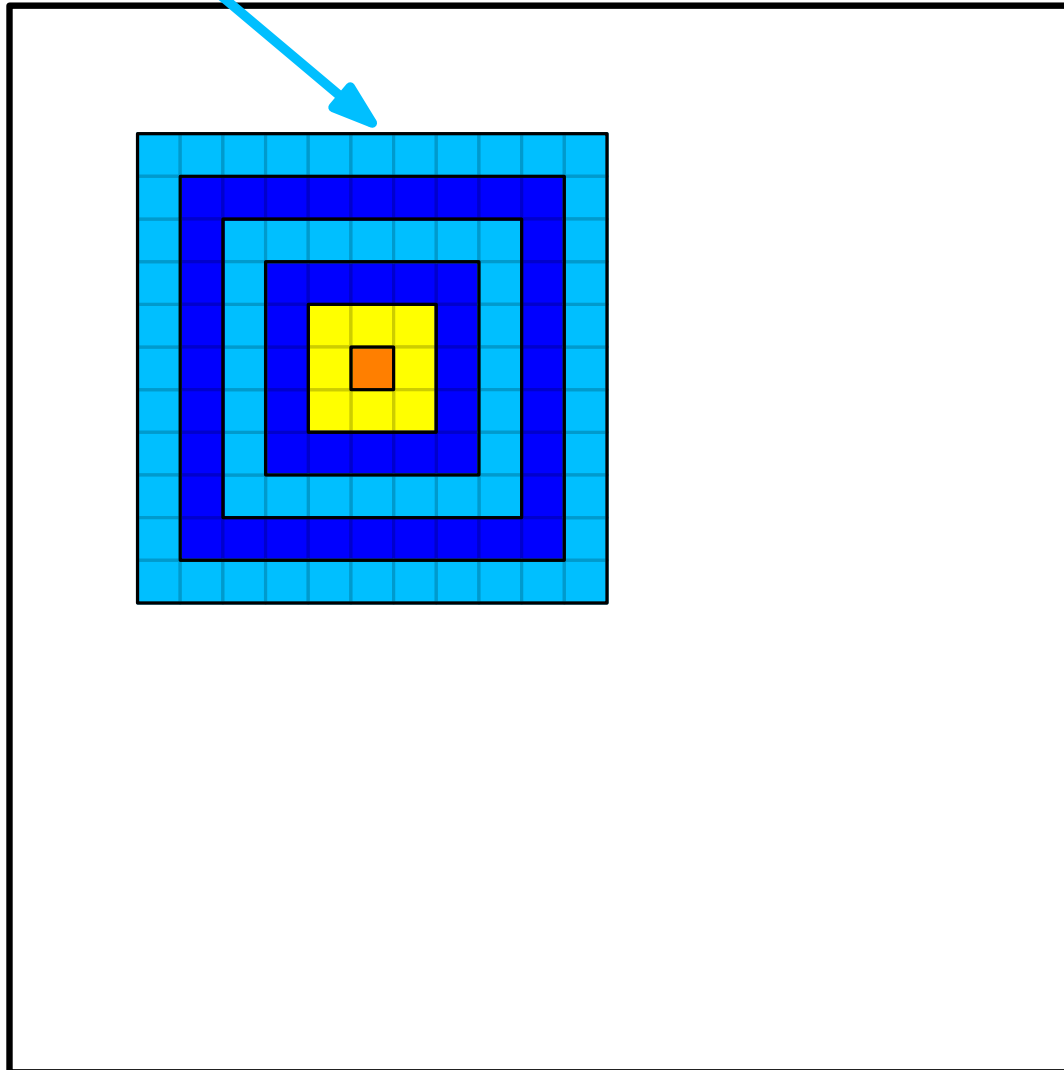
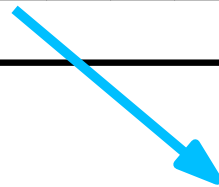
4 or More Colours



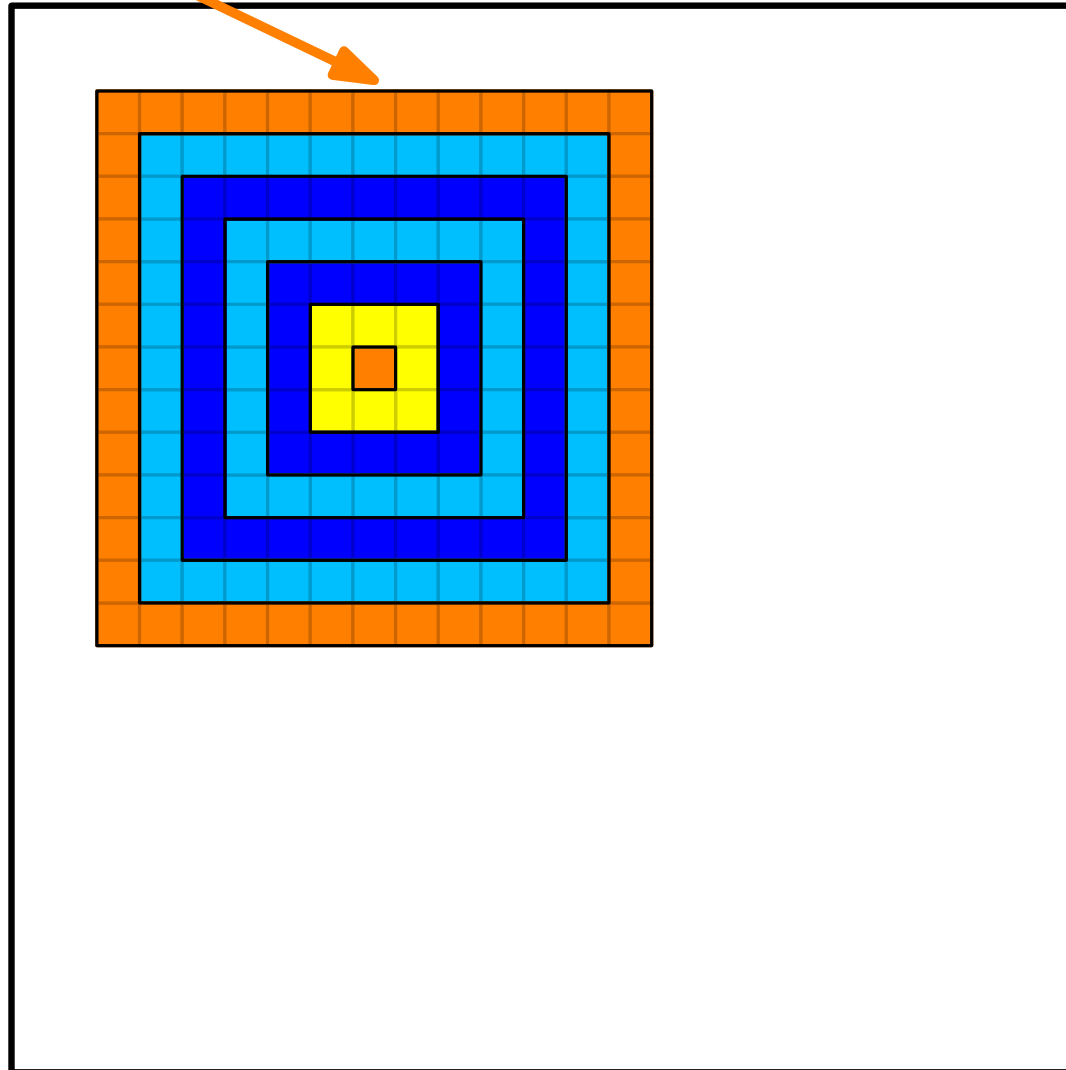
4 or More Colours



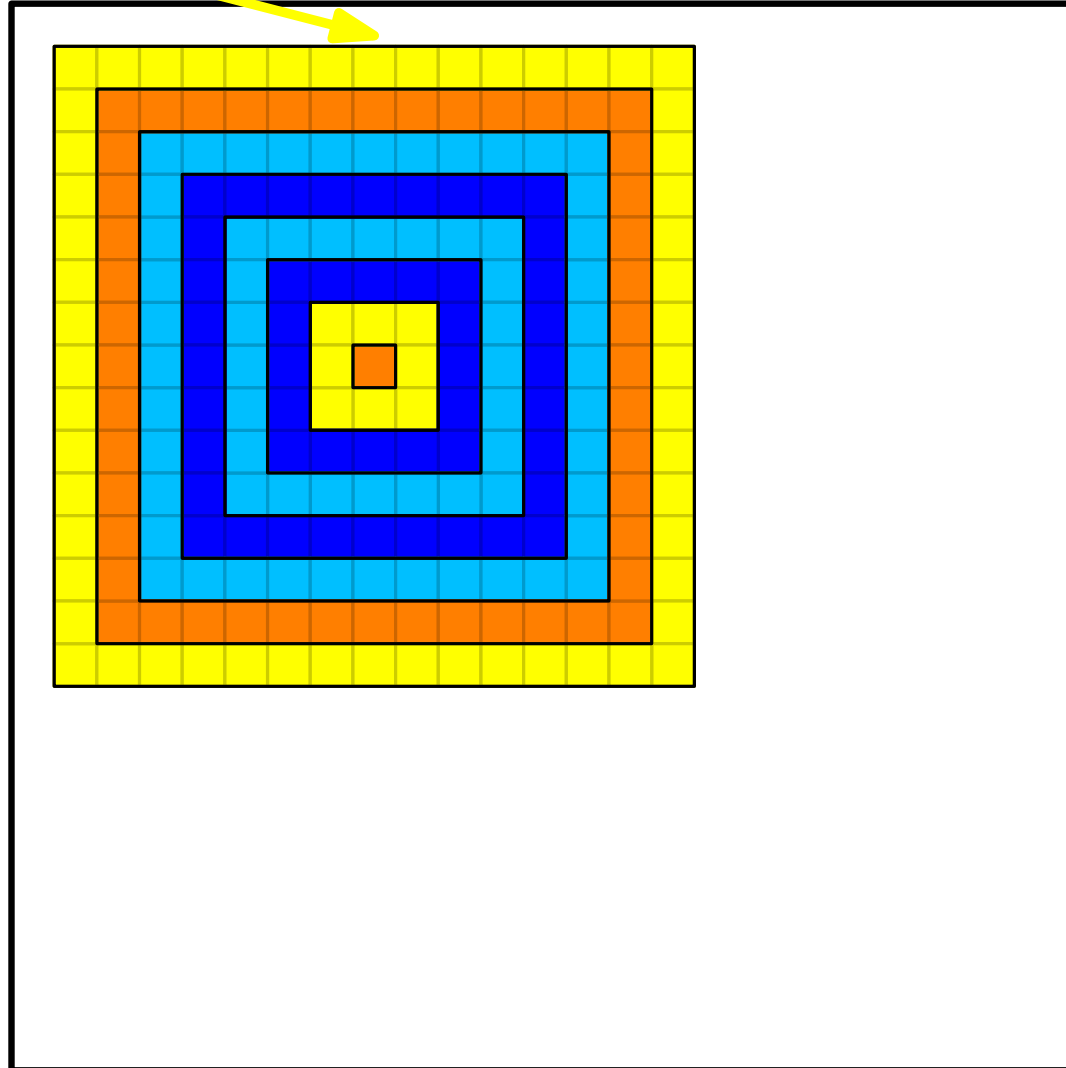
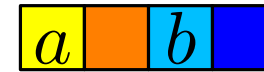
4 or More Colours



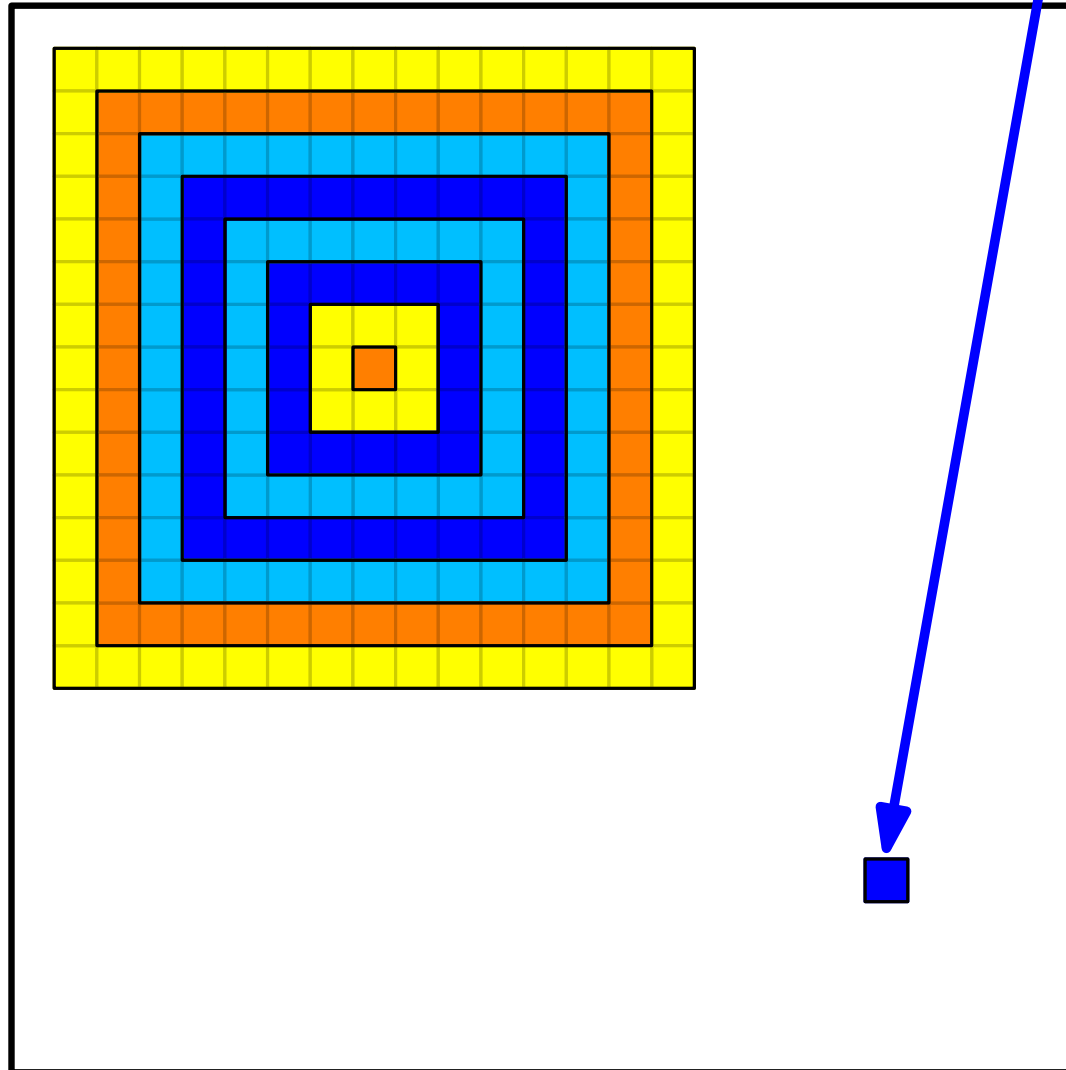
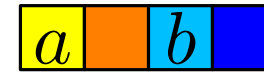
4 or More Colours



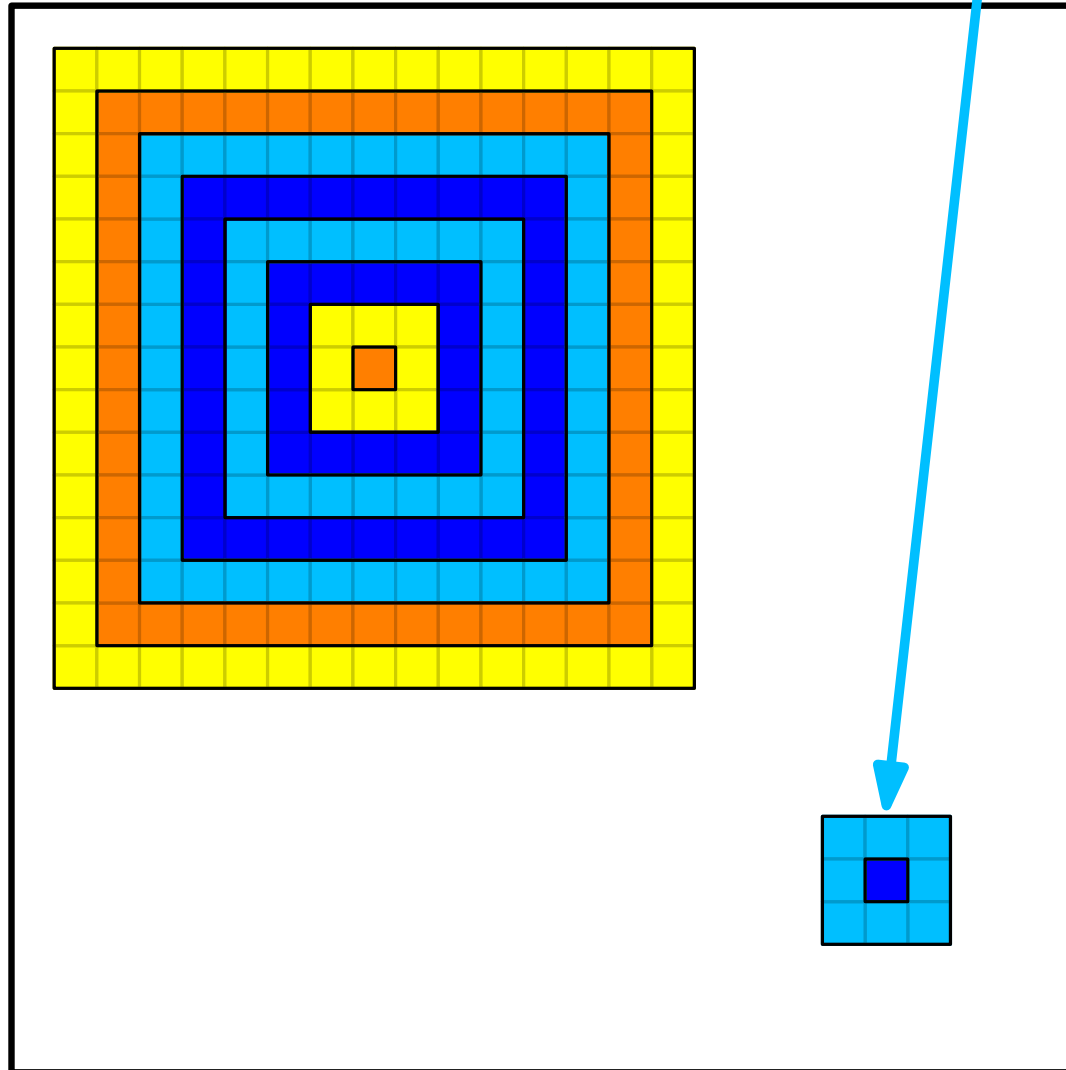
4 or More Colours



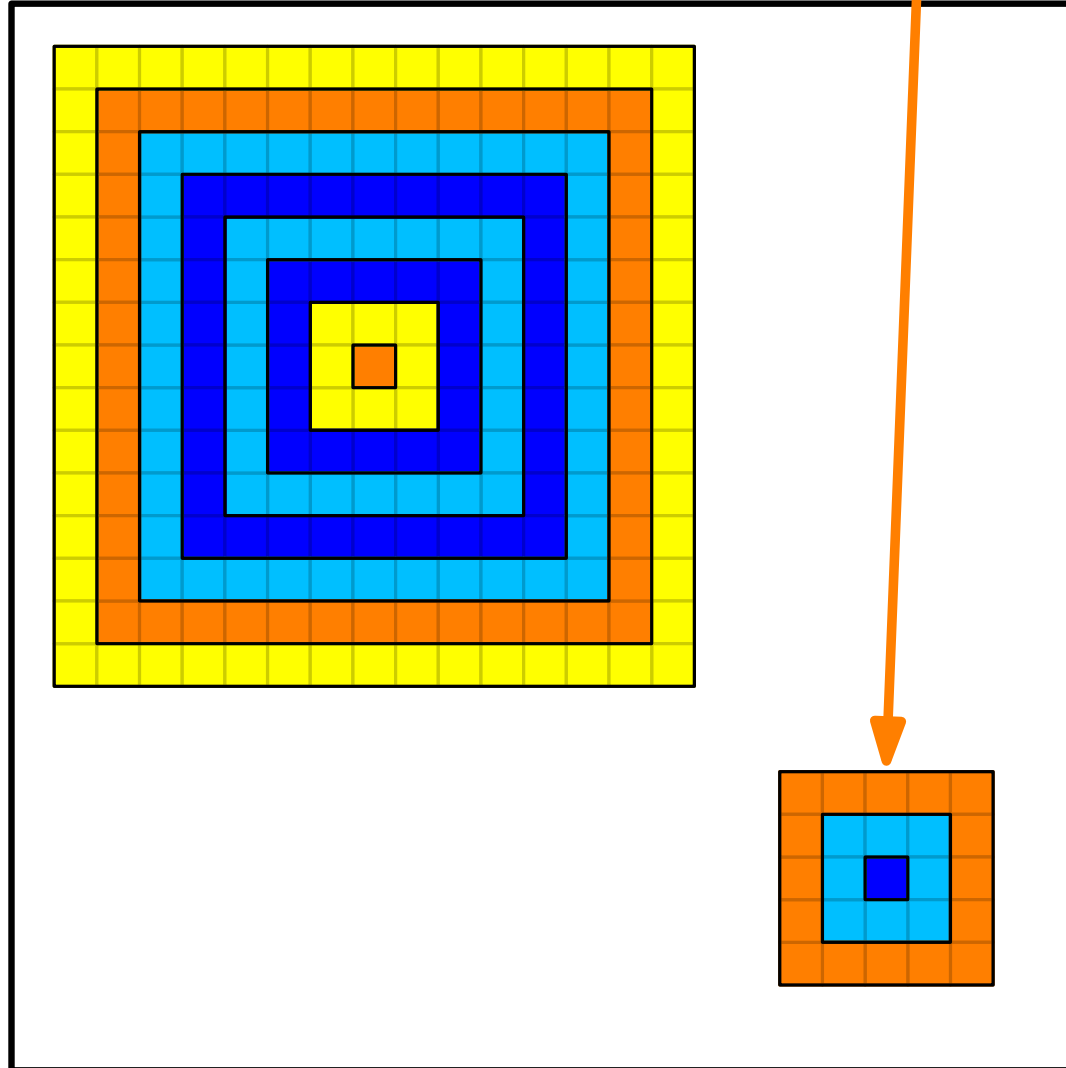
4 or More Colours



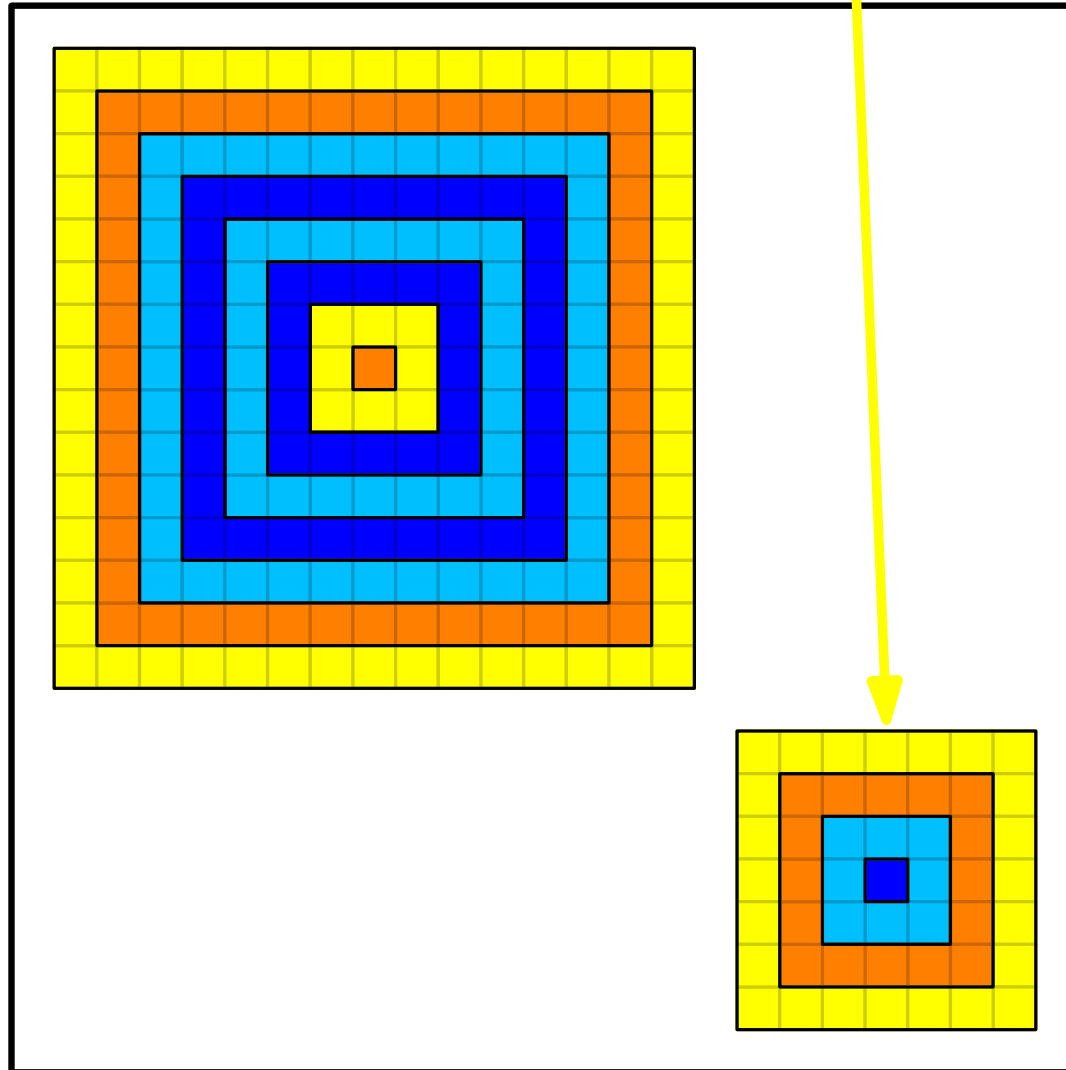
4 or More Colours



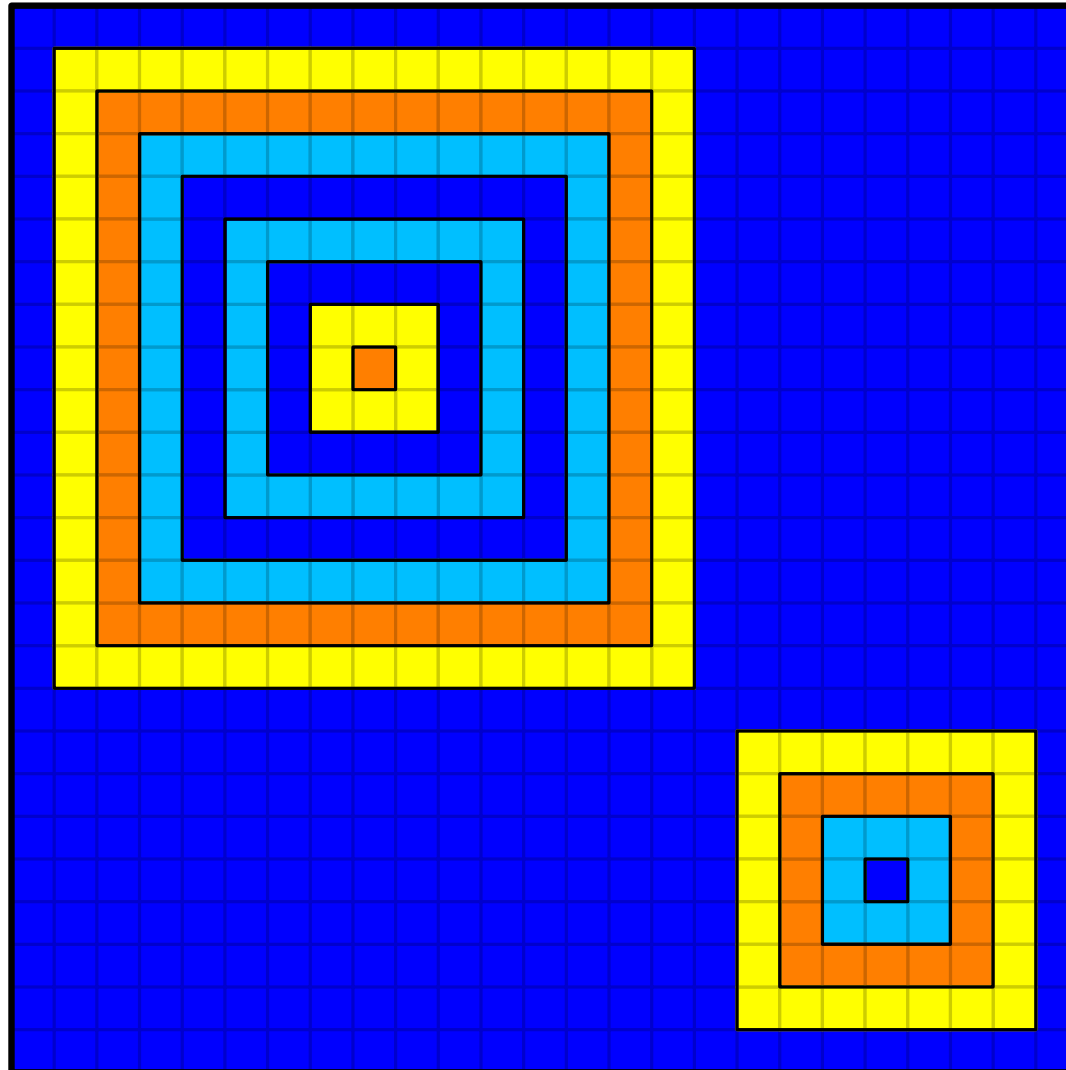
4 or More Colours



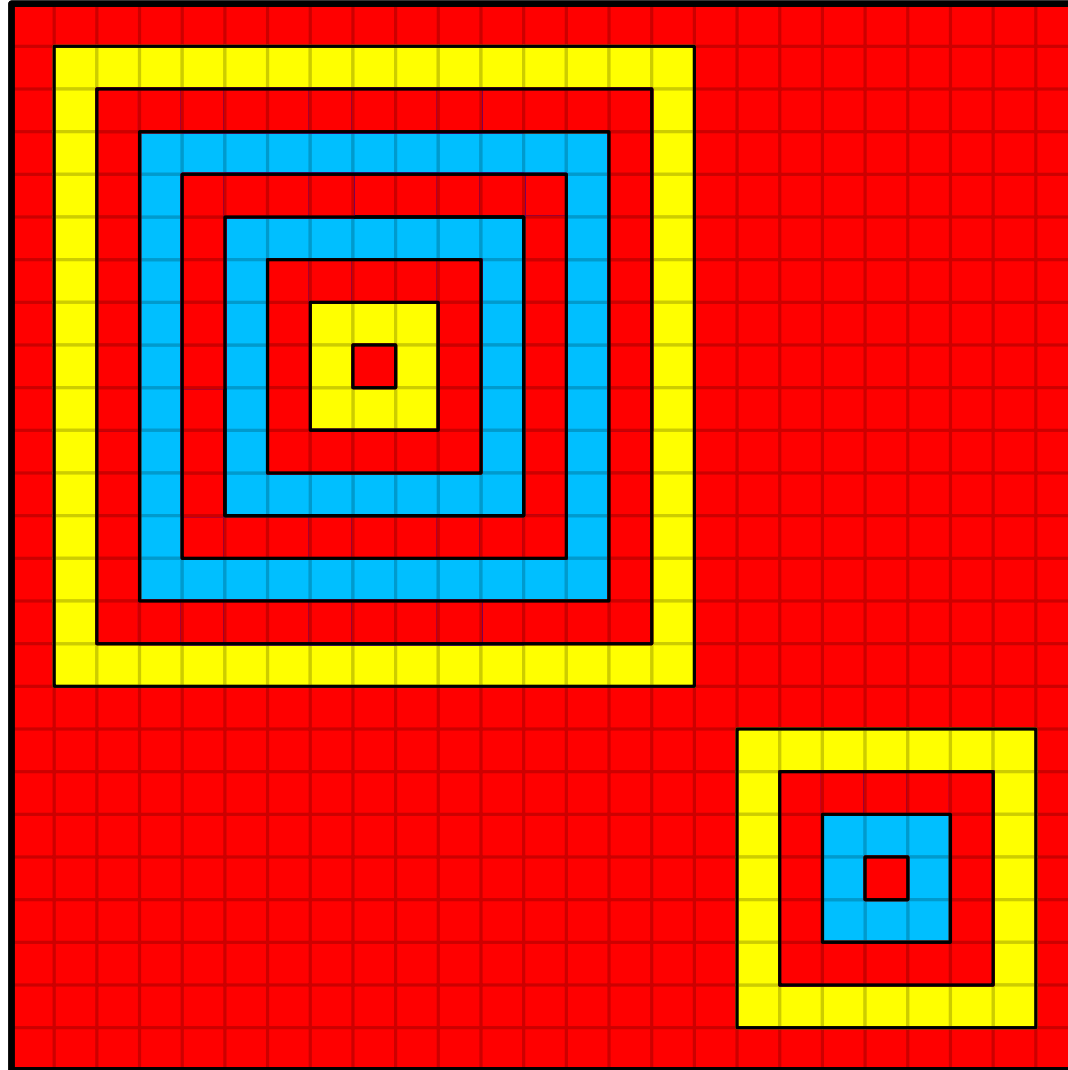
4 or More Colours



4 or More Colours



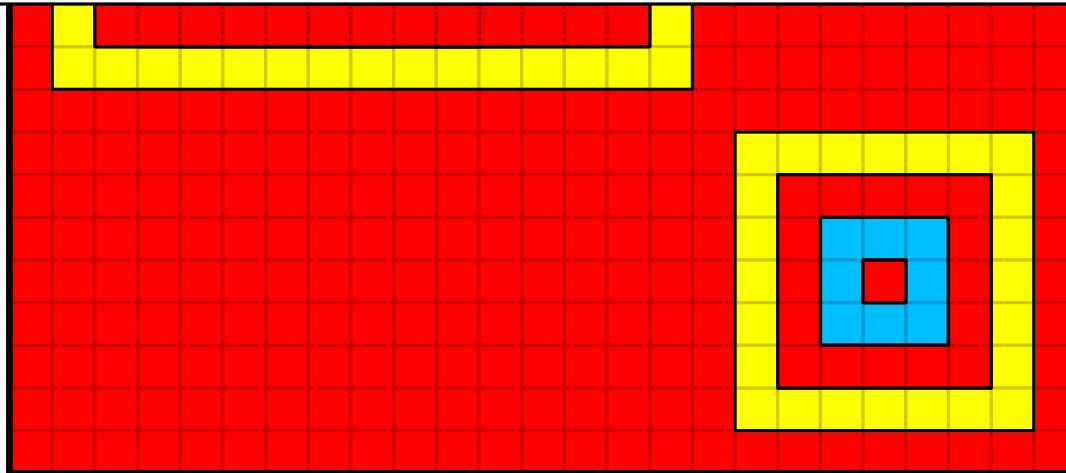
3 Colours



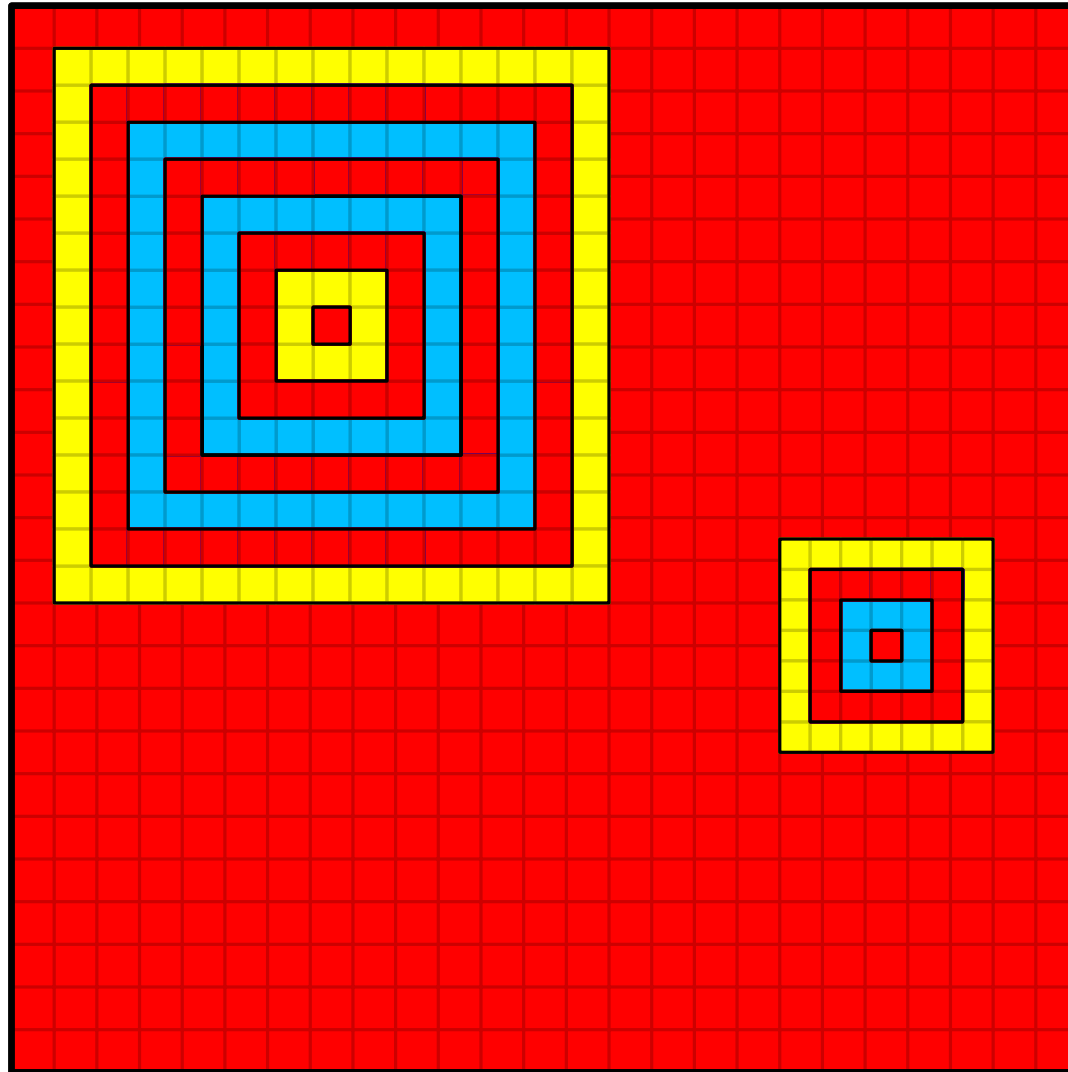
3 Colours



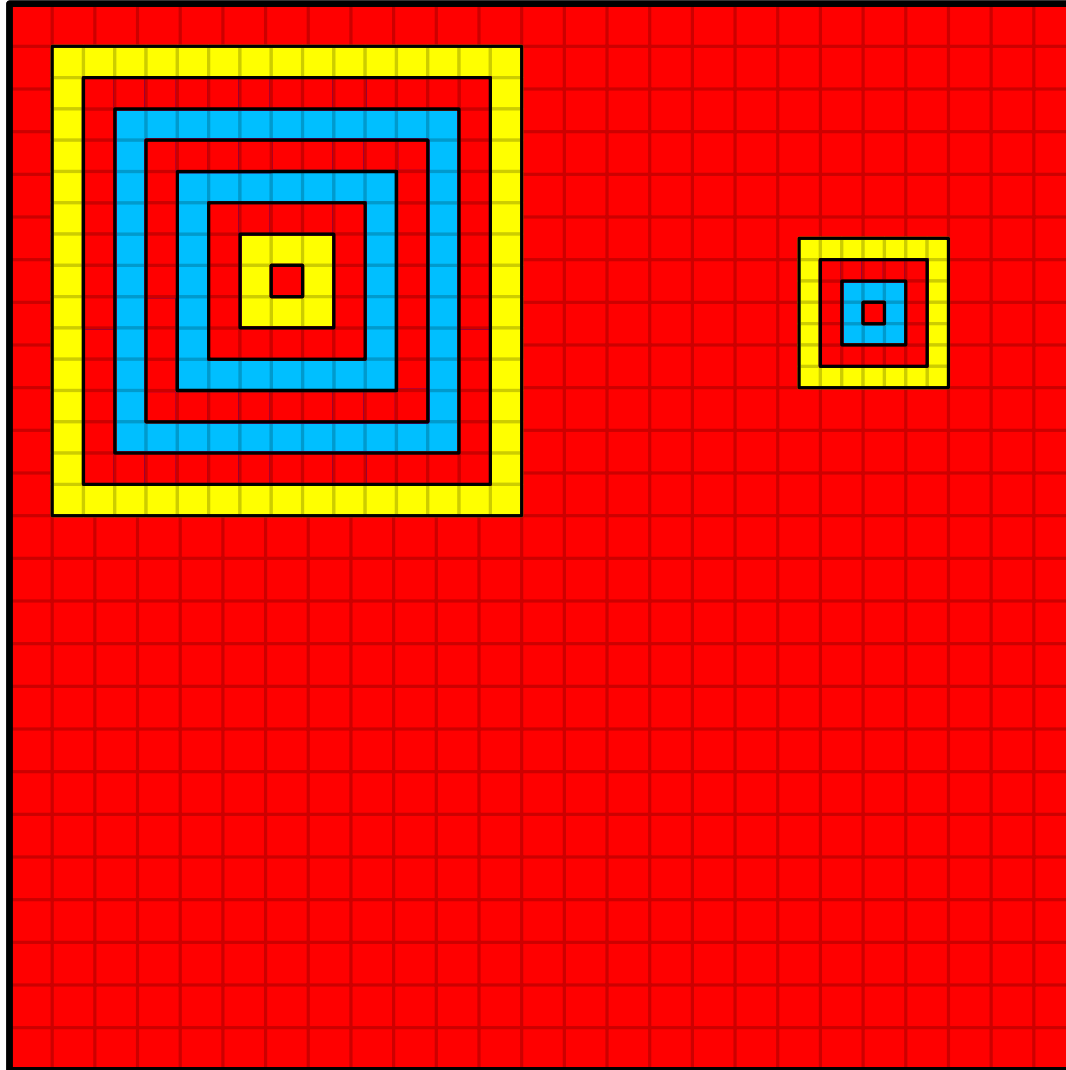
Is there a common supersequence of length at most 4?



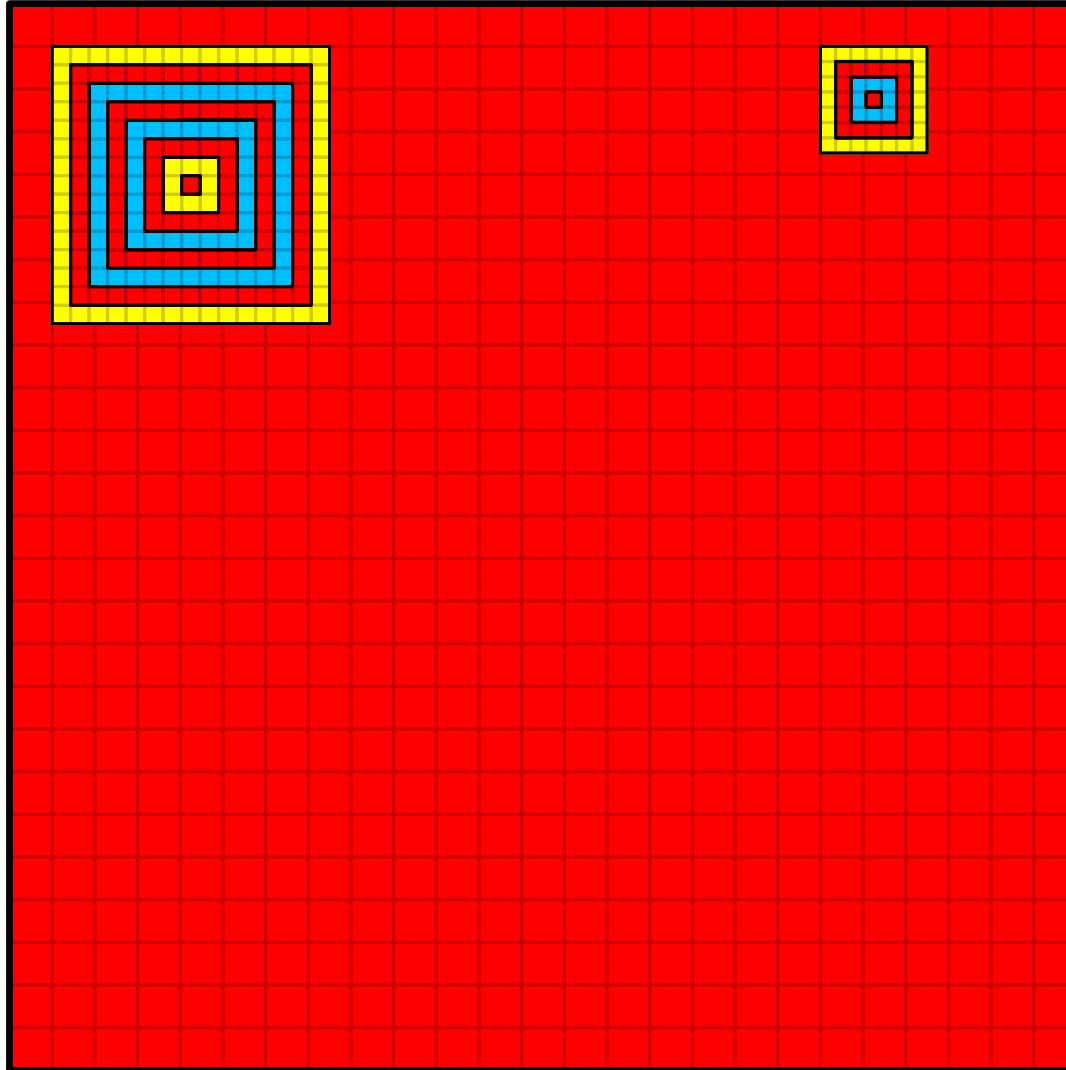
3 Colours



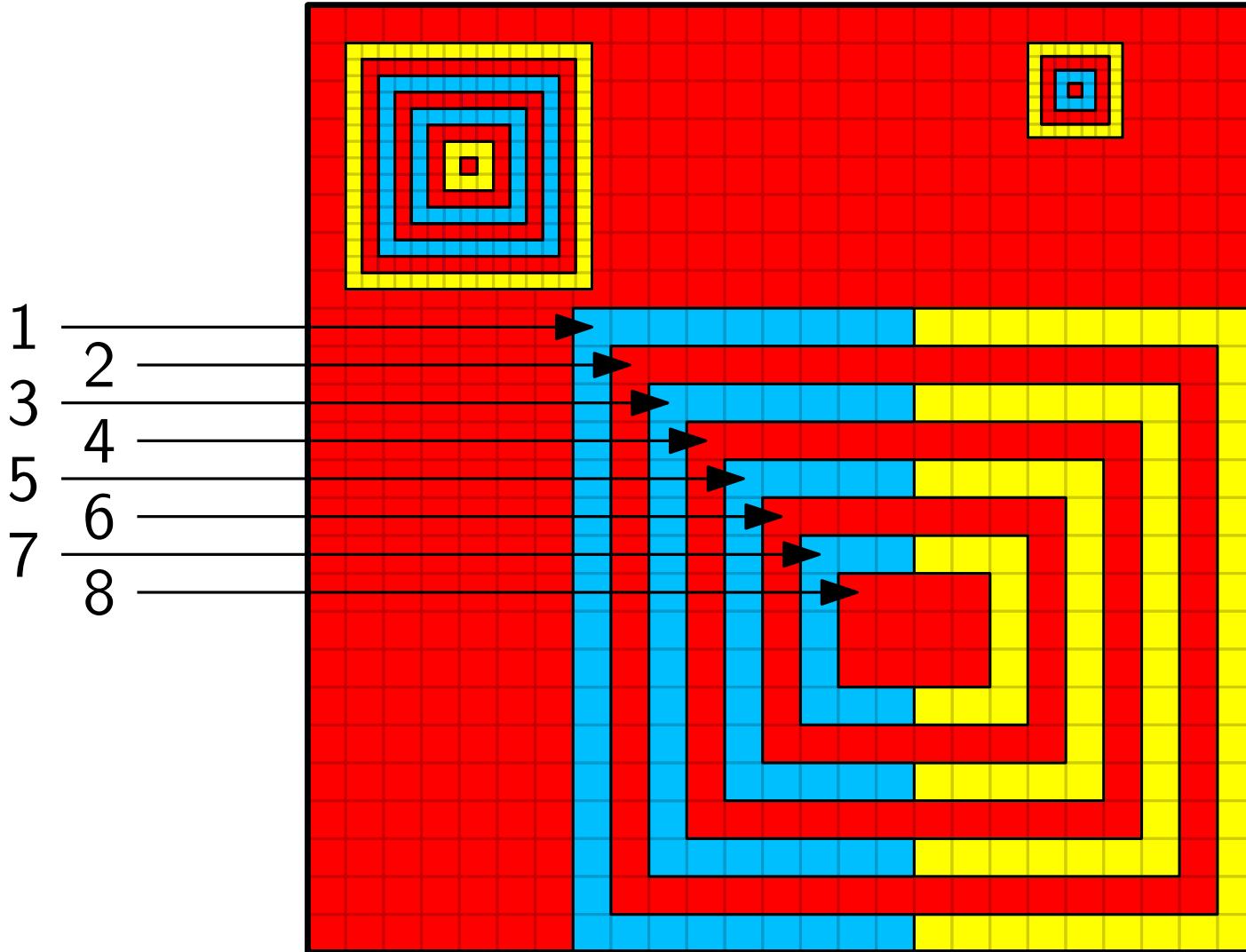
3 Colours



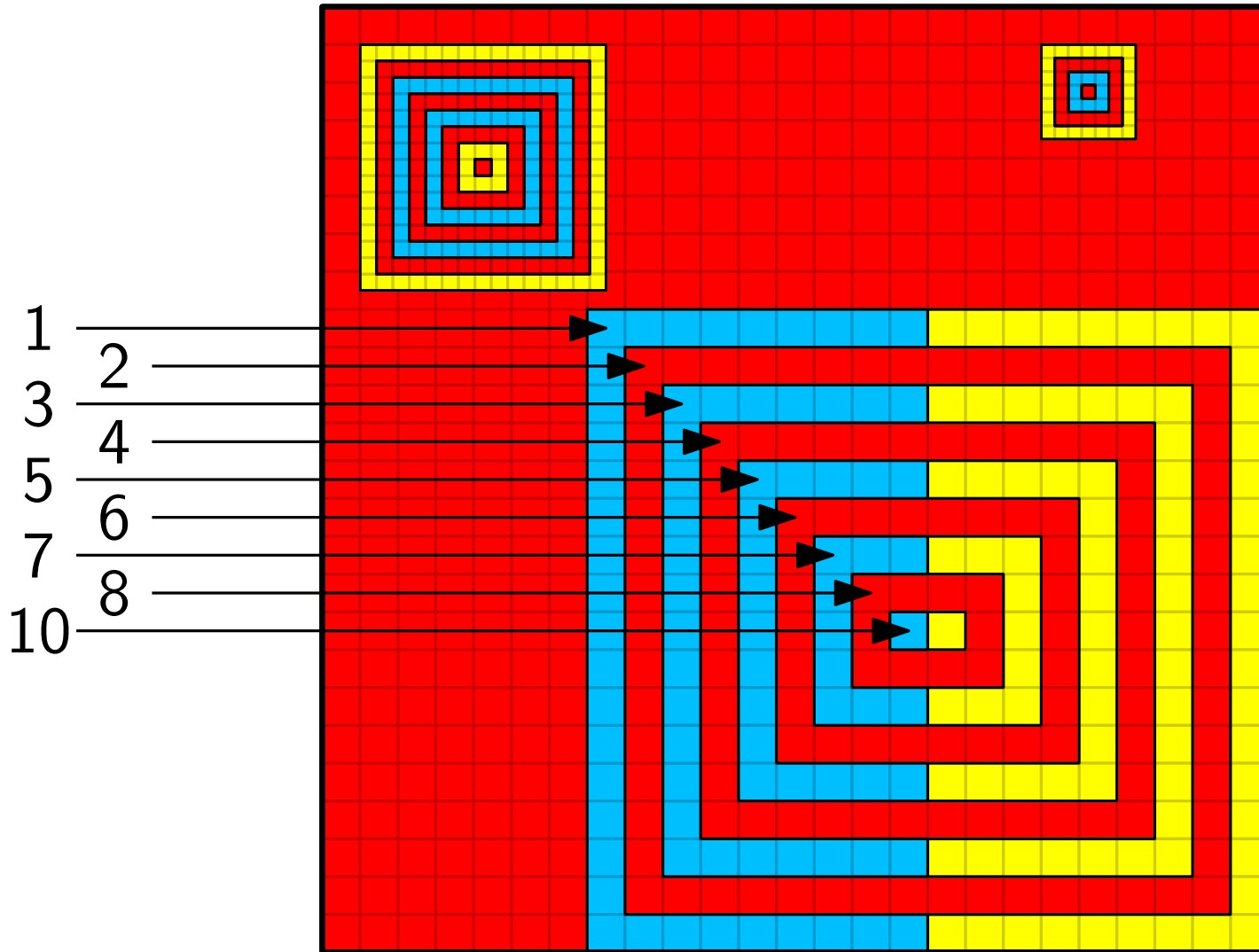
3 Colours



3 Colours



3 Colours



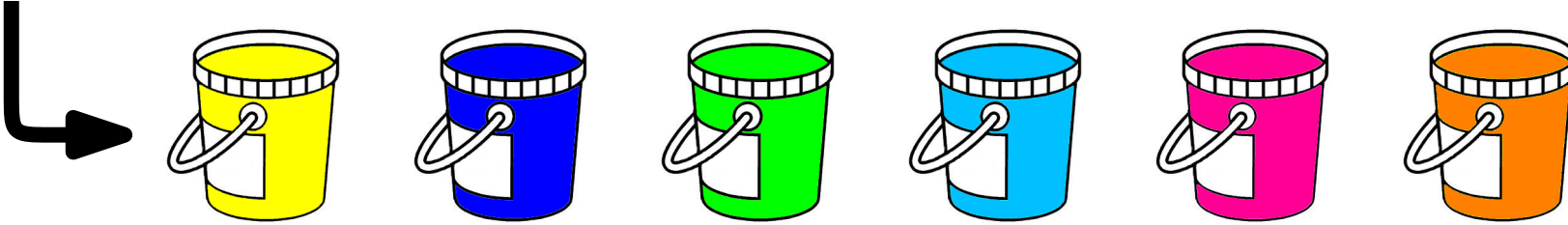
Approximation



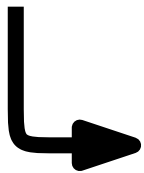
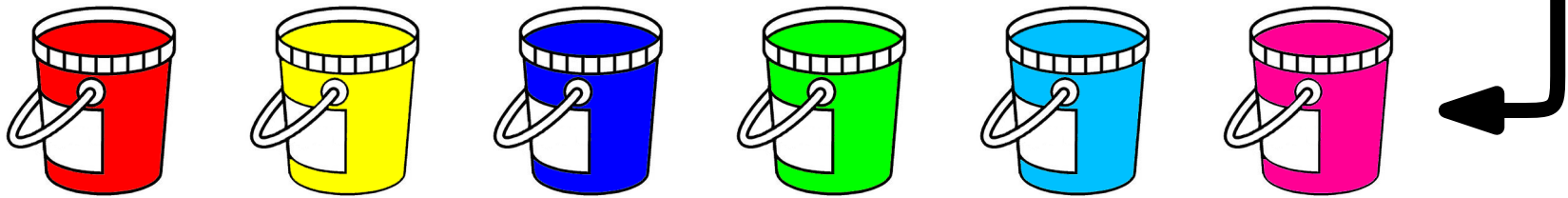
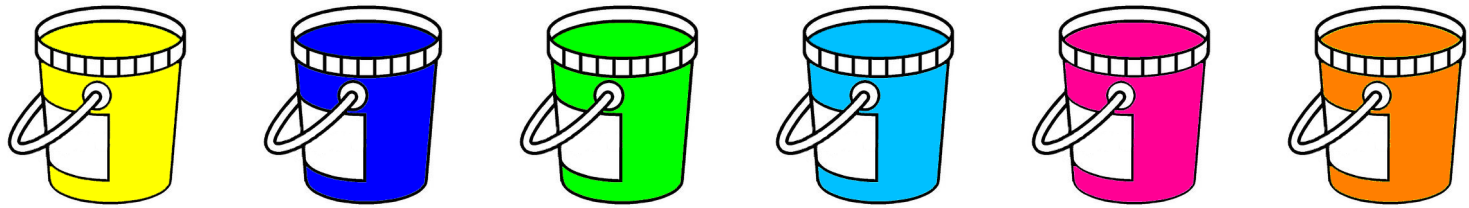
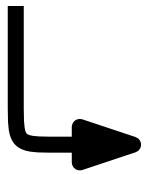
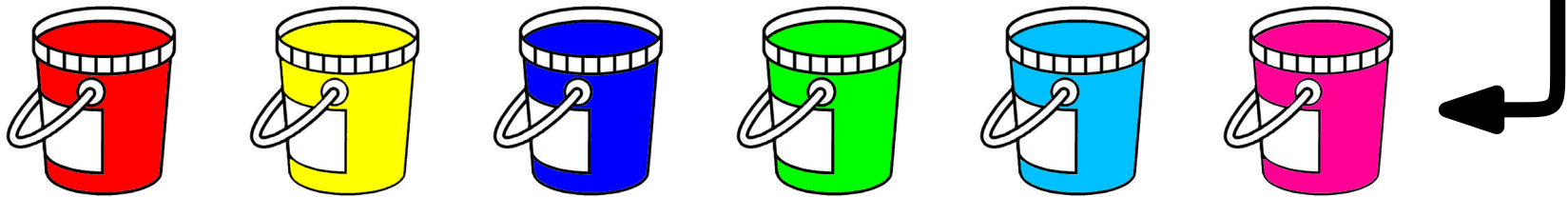
Approximation



Approximation

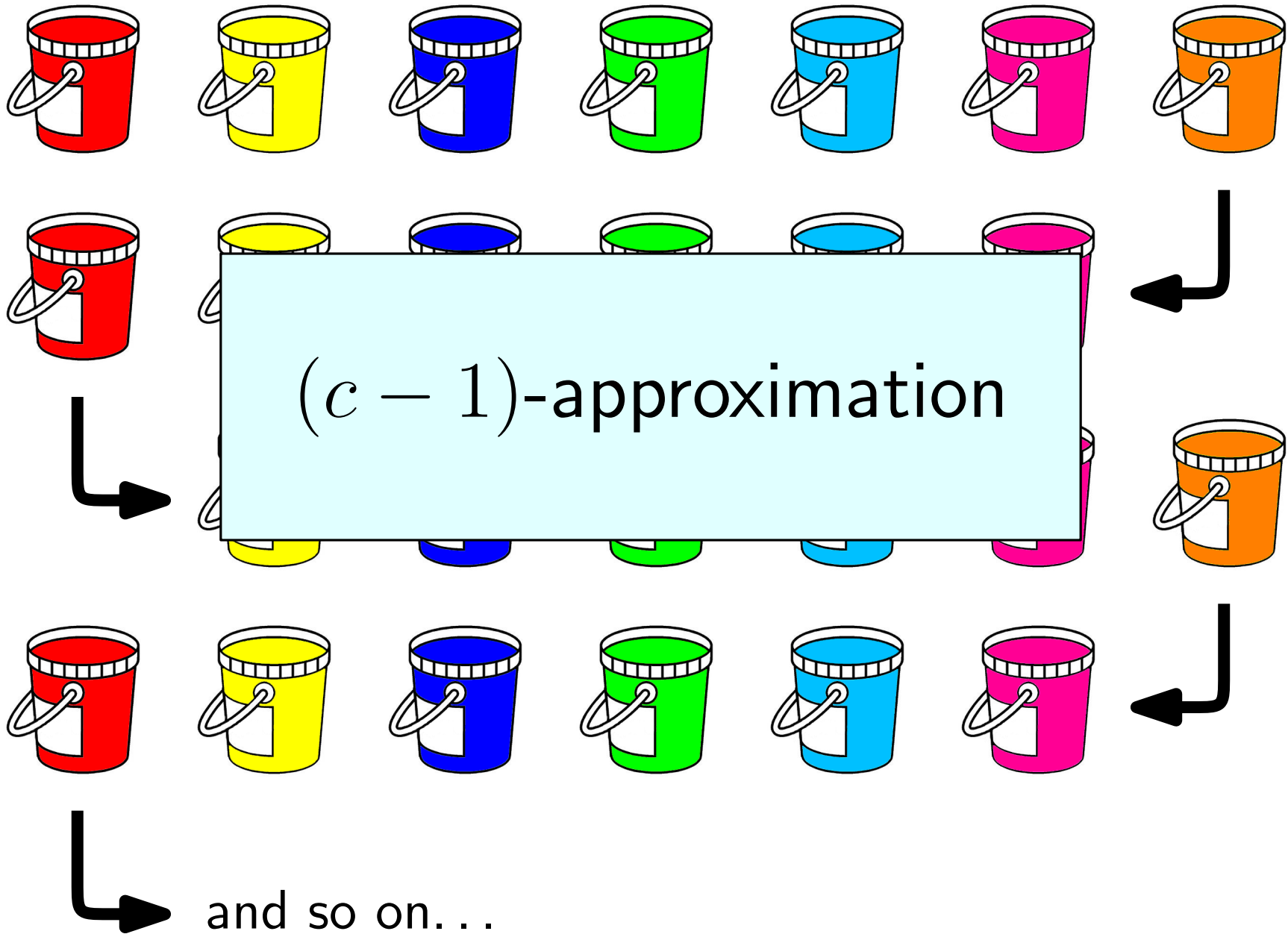


Approximation



and so on...

Approximation



Randomised Approximation

Flooding sequence:



Randomised Approximation

Flooding sequence:



Shuffle

Randomised Approximation

Flooding sequence:



Shuffle



Randomised Approximation

Flooding sequence:



Probability $\frac{1}{2}$



Shuffle

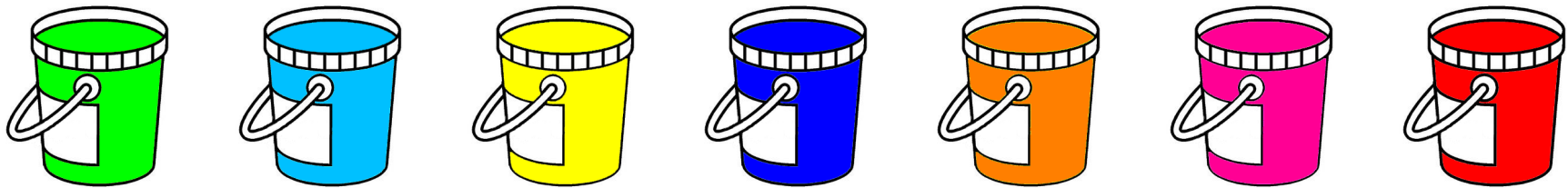


Randomised Approximation

Flooding sequence:



Probability $\frac{1}{2}$



Shuffle

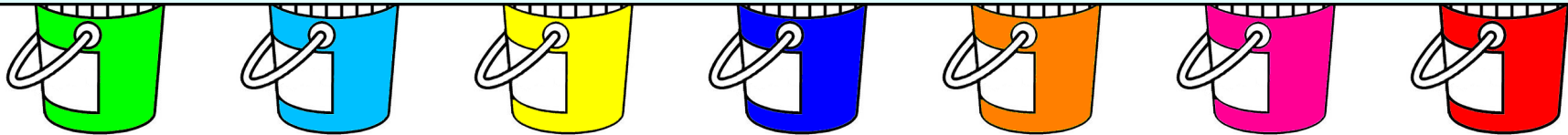


Randomised Approximation

Flooding sequence:



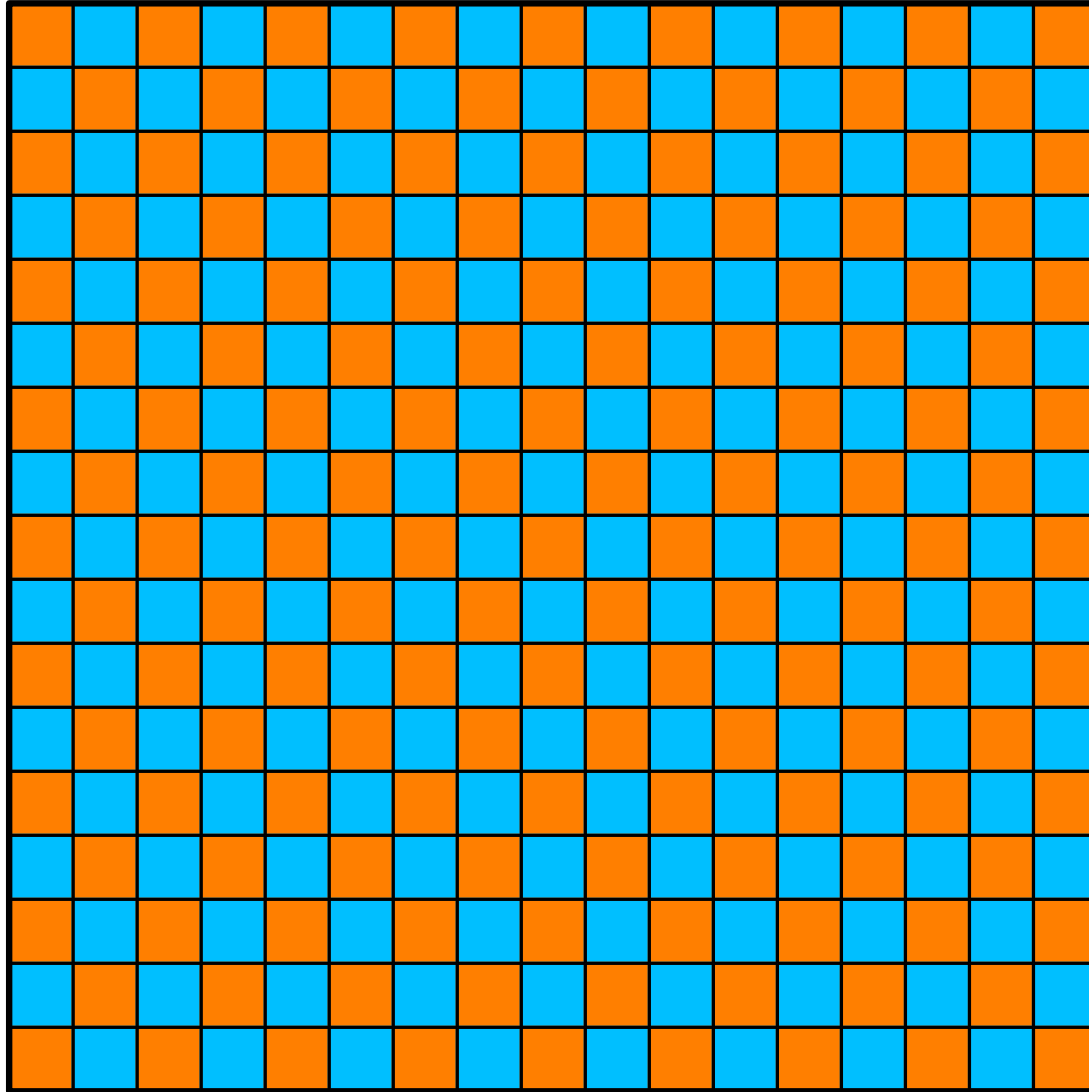
Randomised $(2c/3)$ -approximation



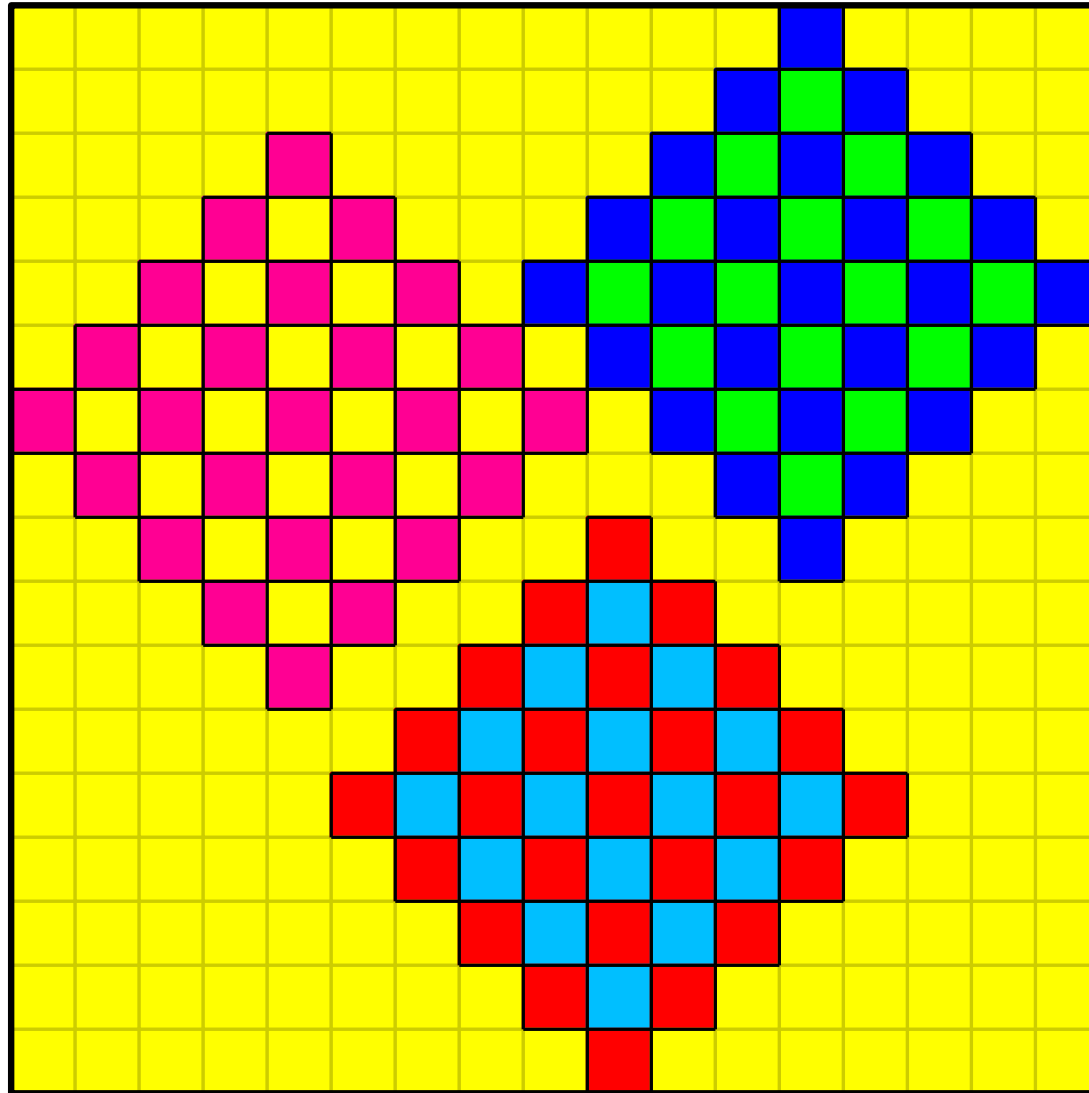
Shuffle



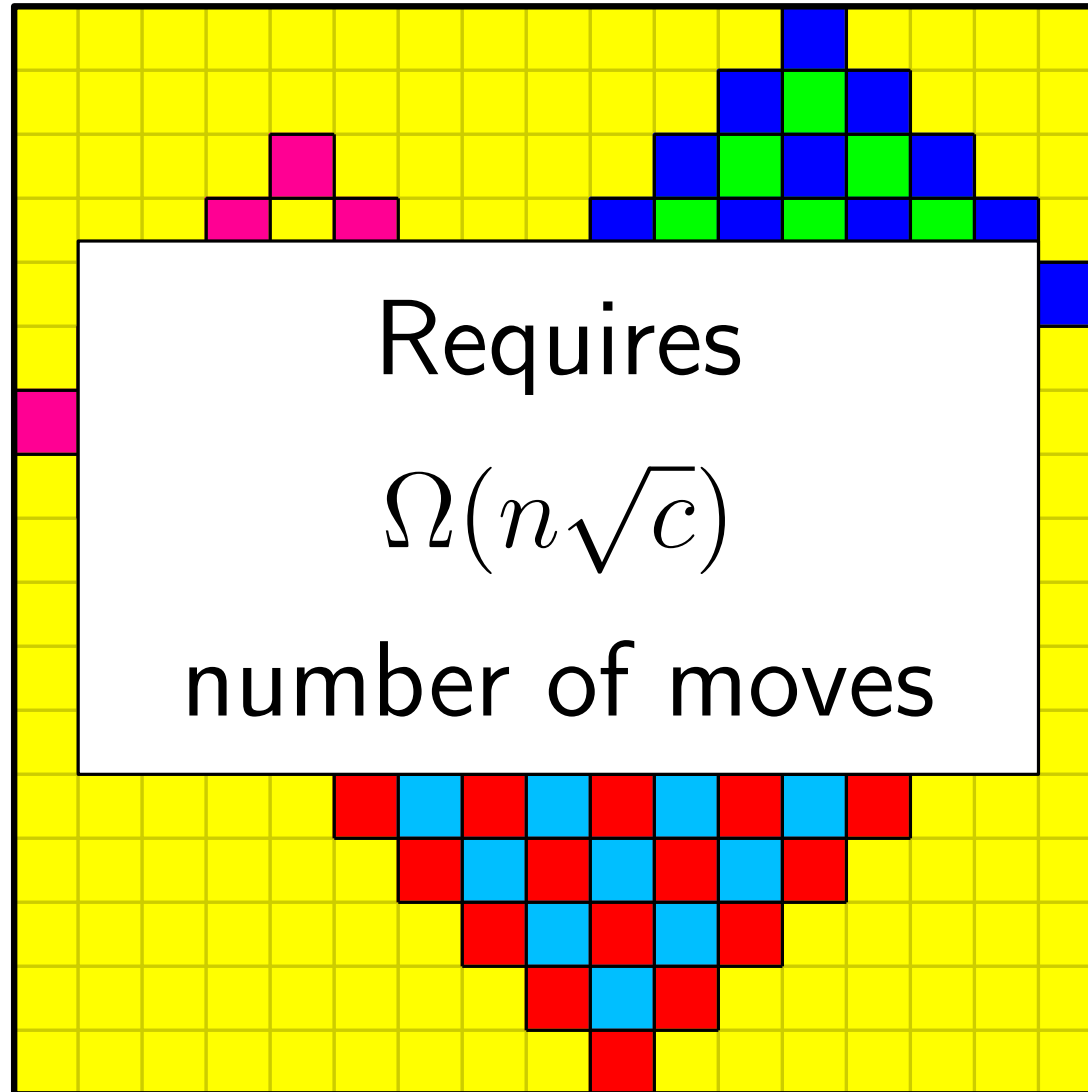
Bad Boards



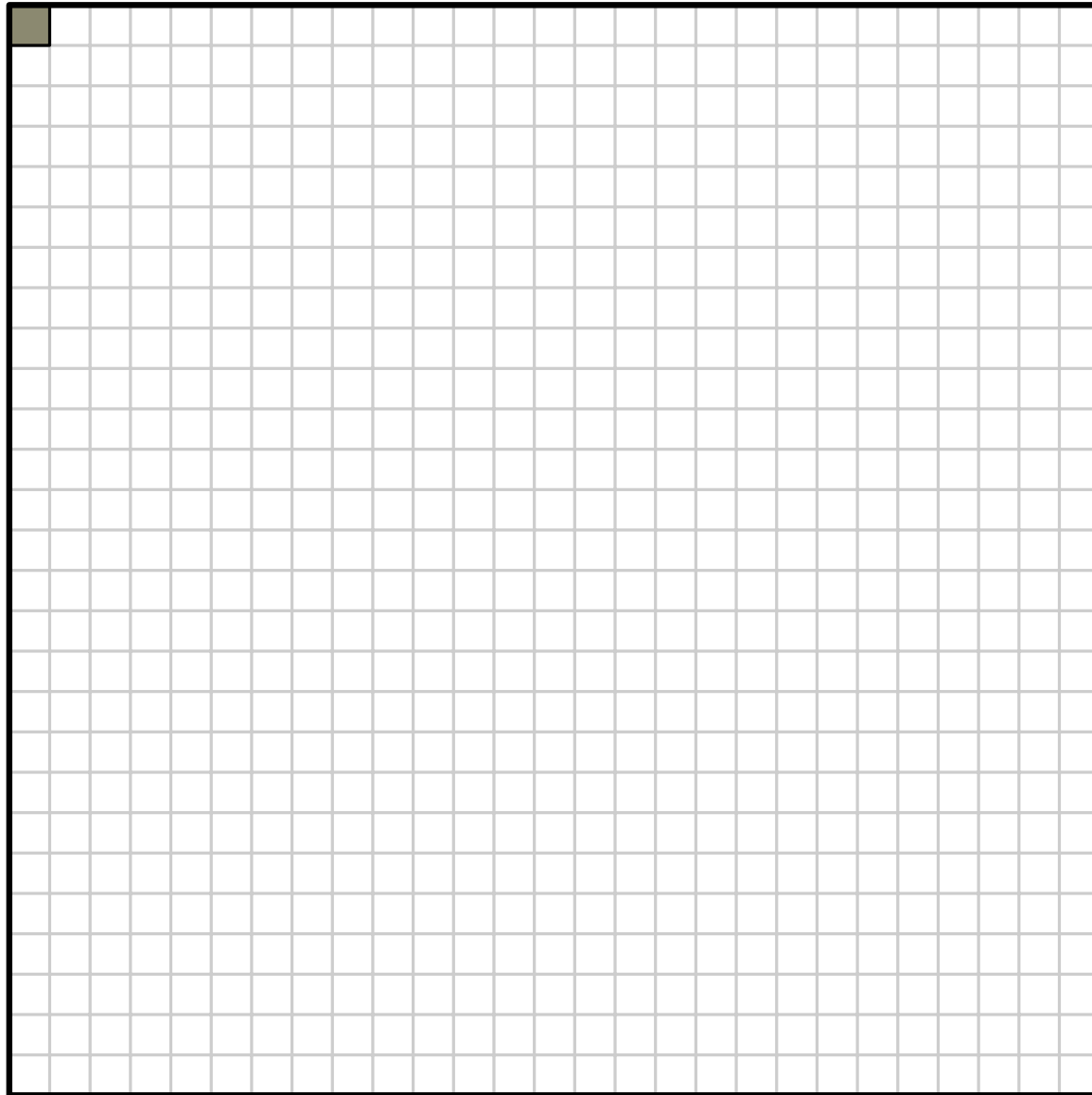
Bad Boards



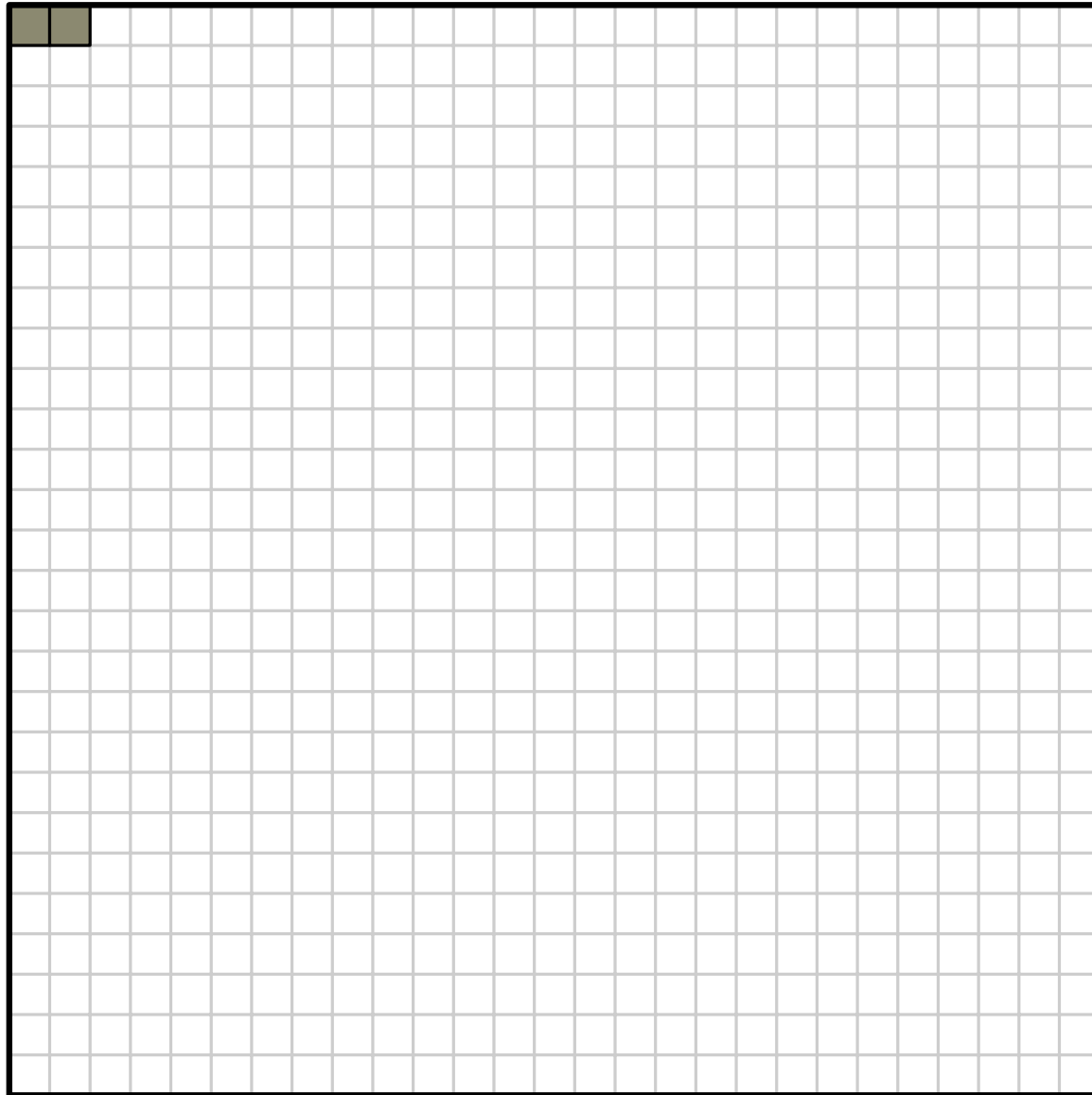
Bad Boards



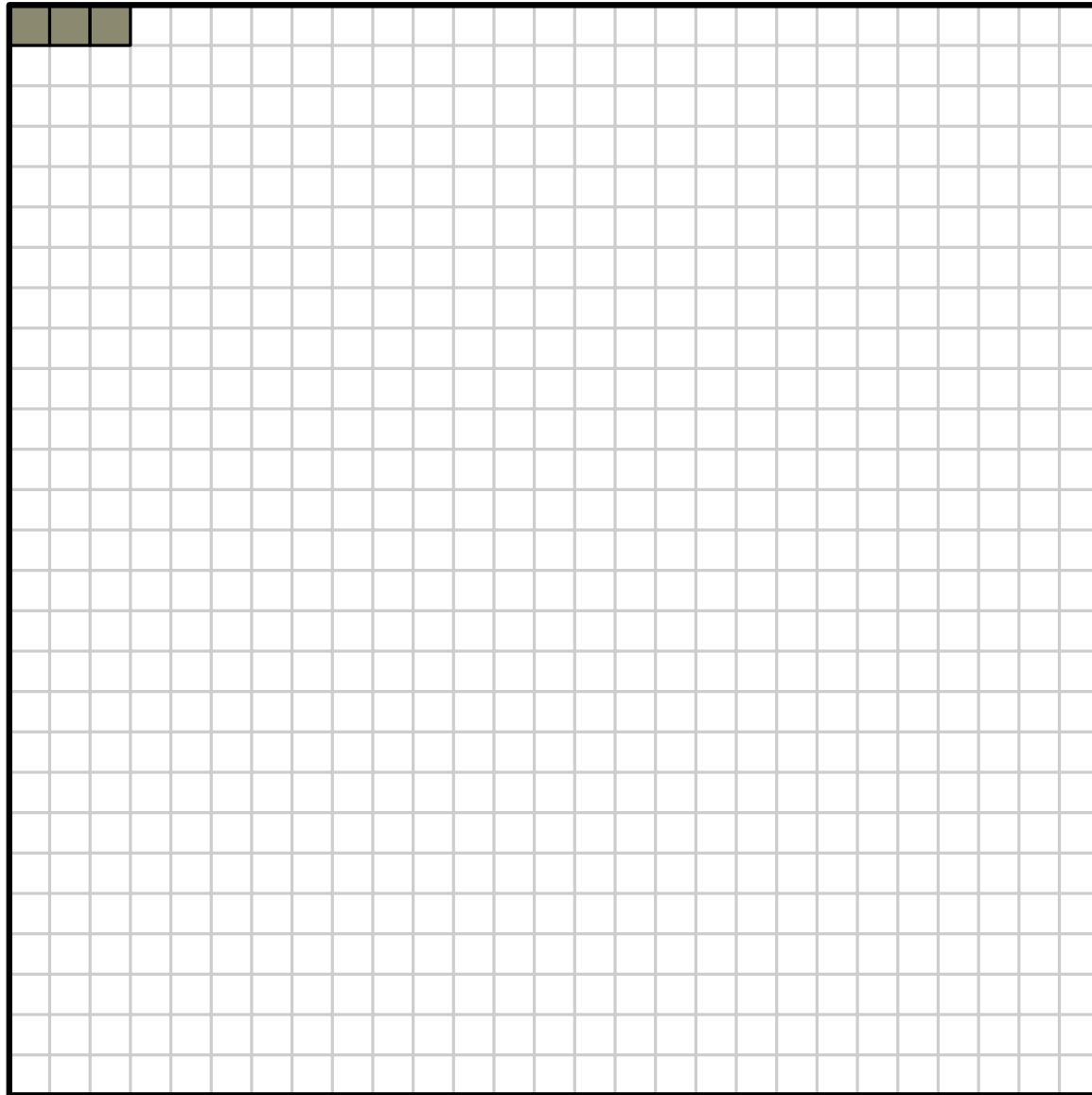
Upper Bound



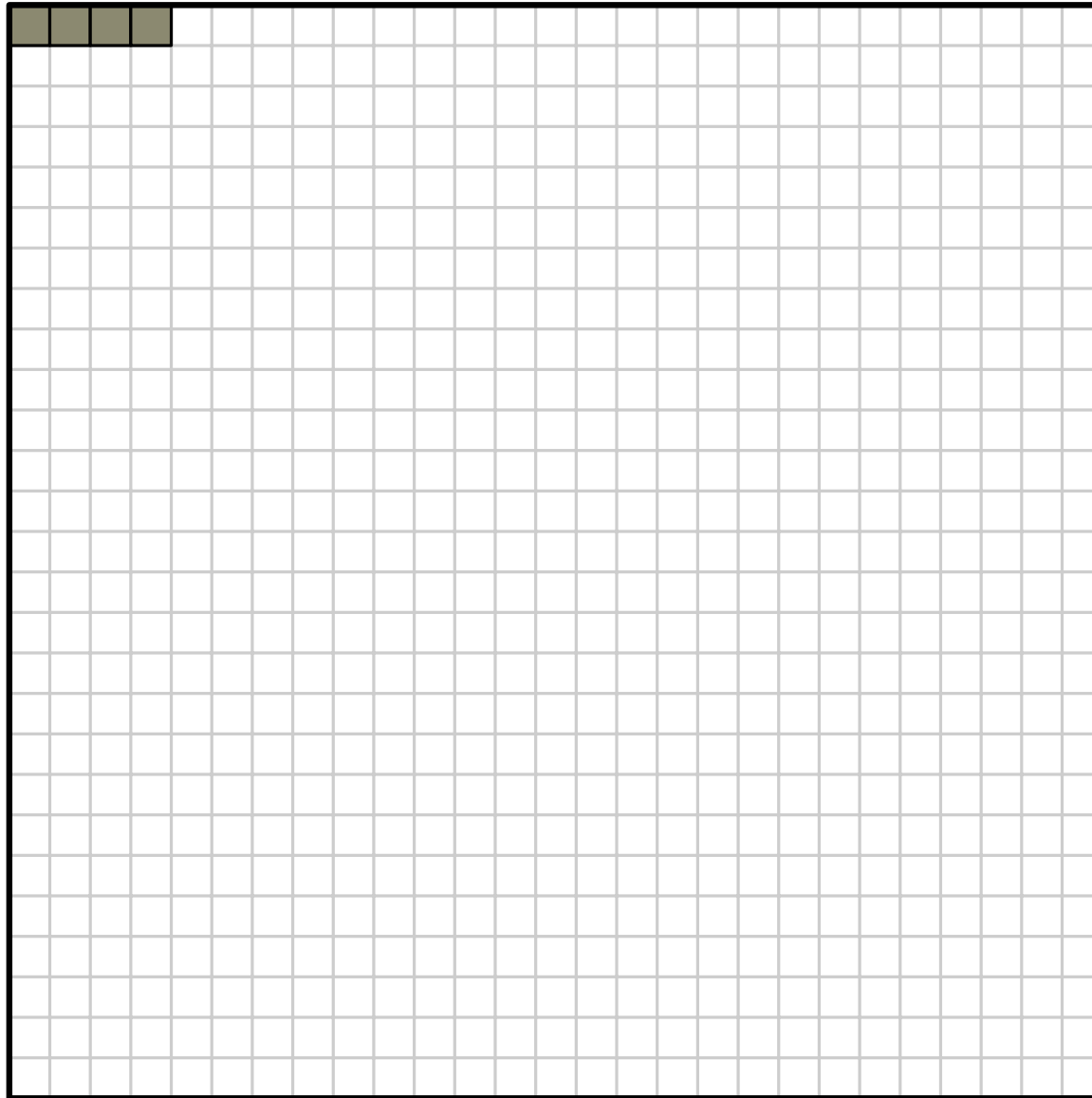
Upper Bound



Upper Bound

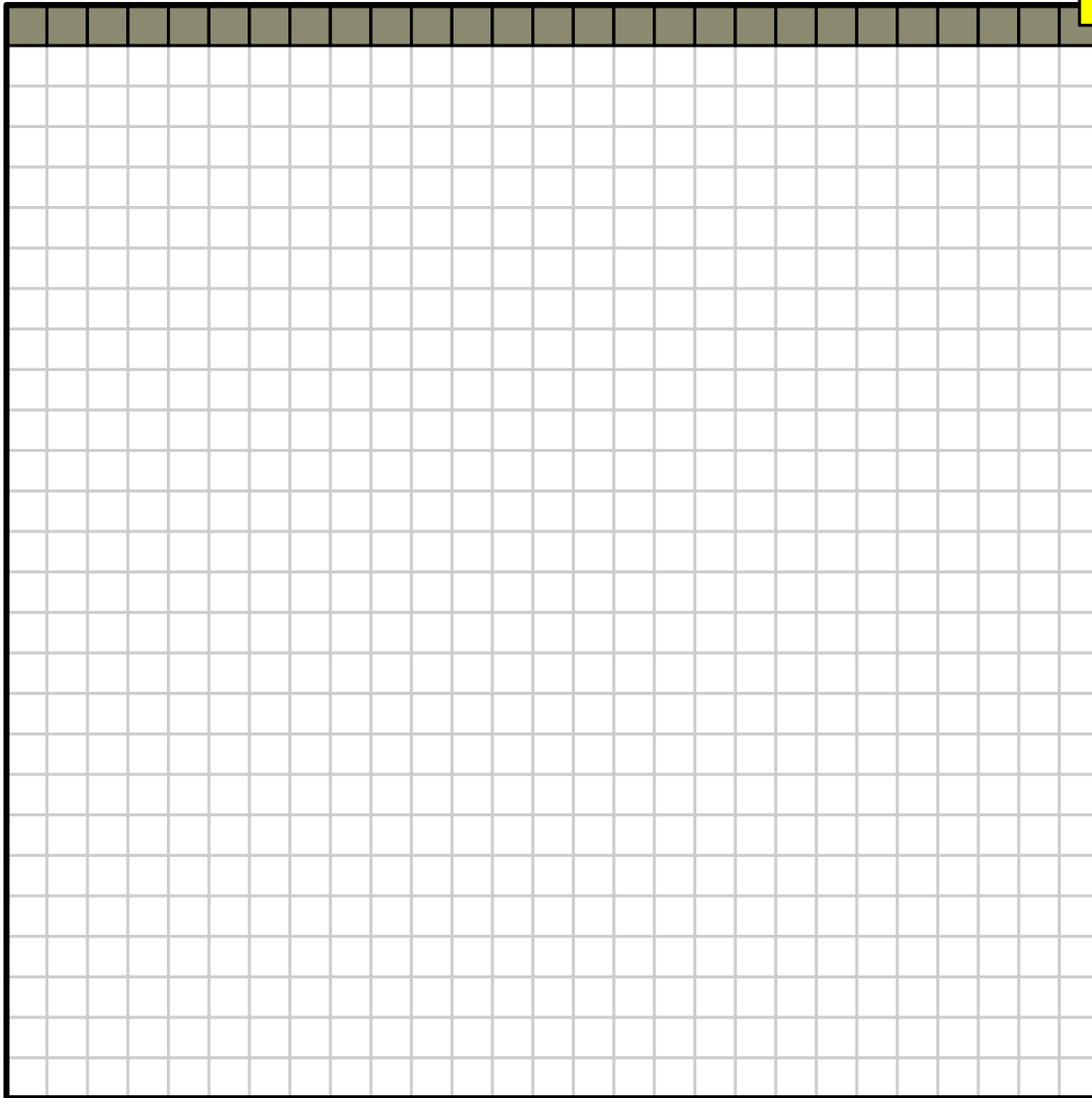


Upper Bound



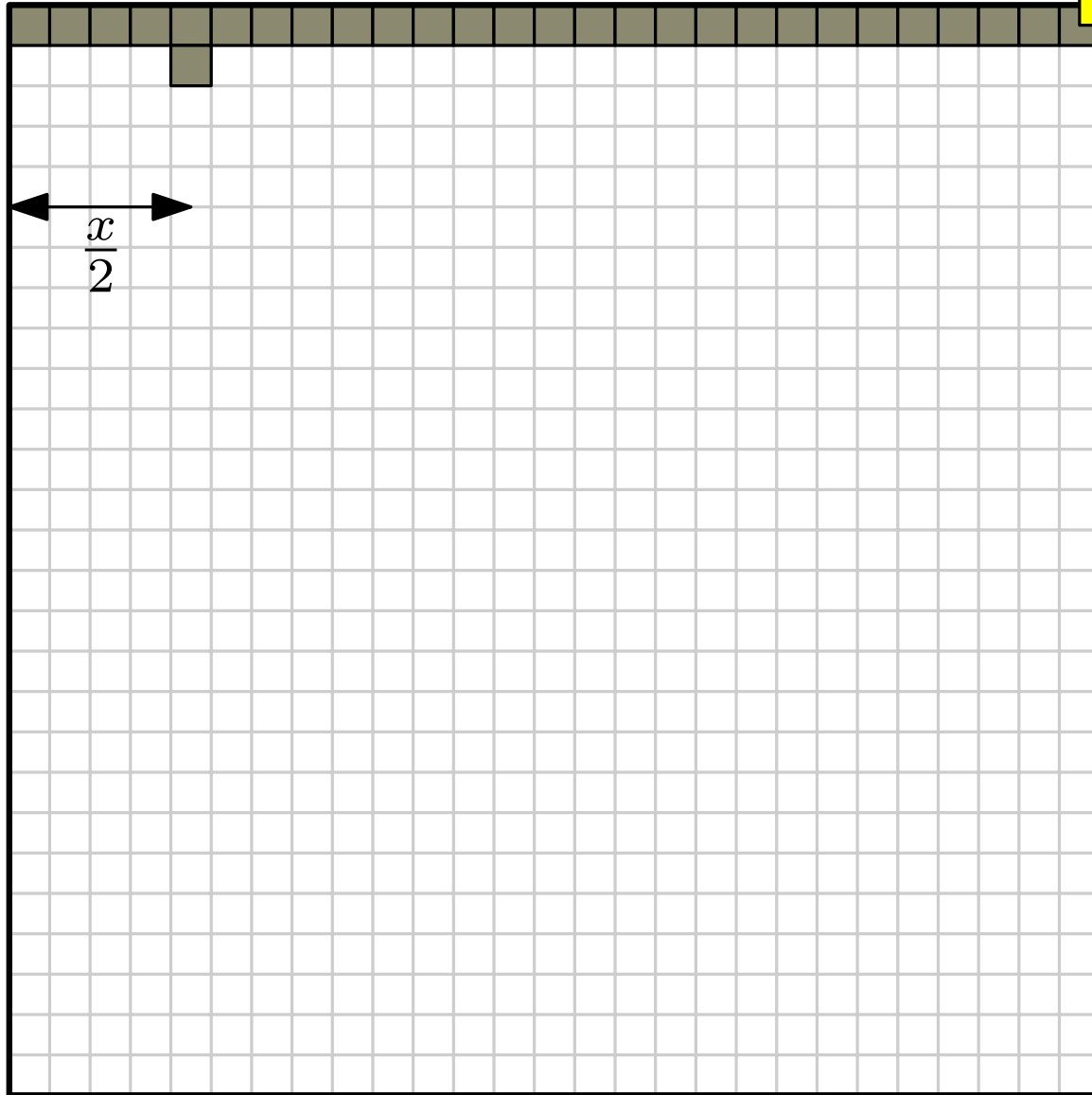
Upper Bound

$$n - 1$$



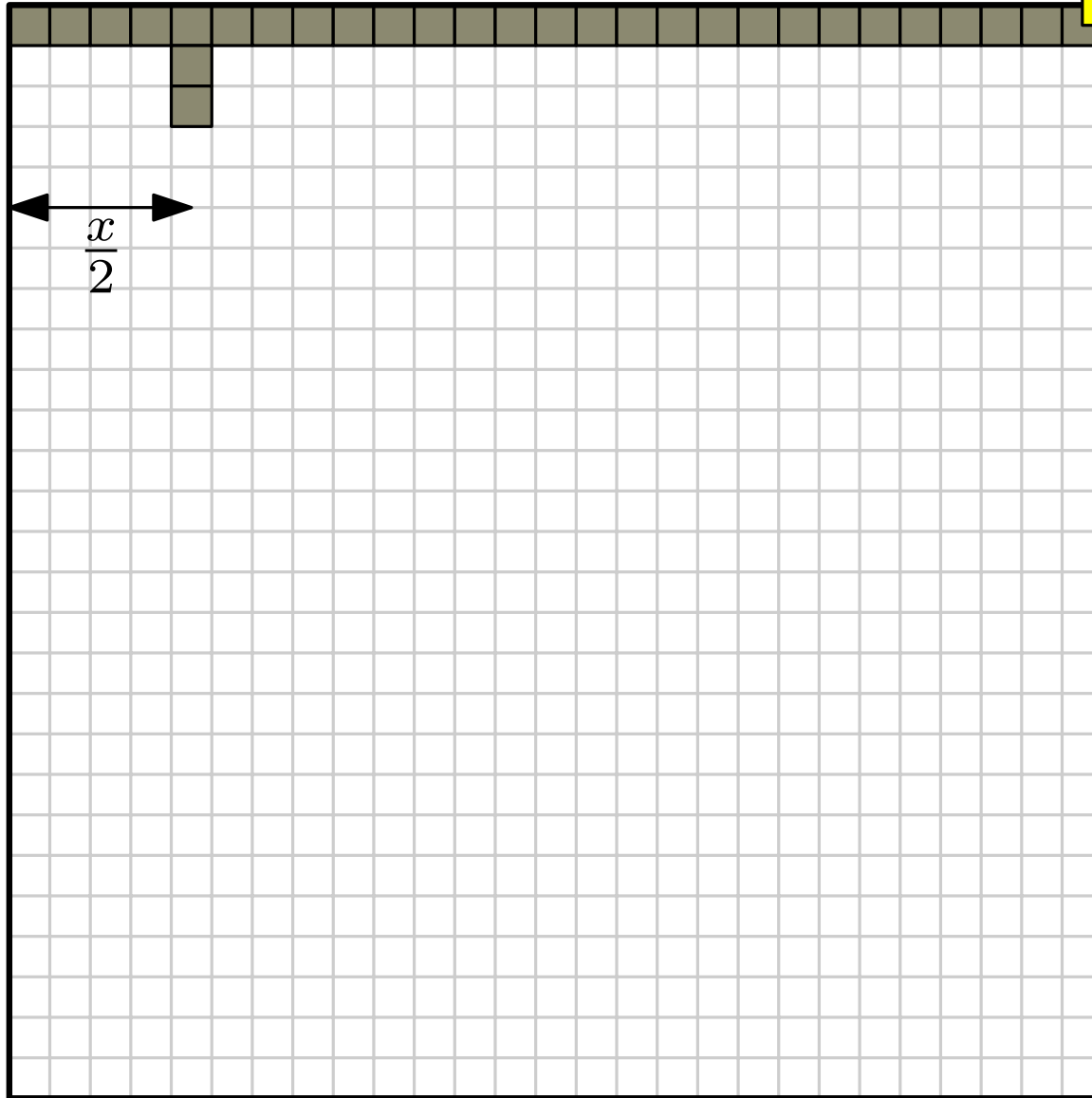
Upper Bound

$n - 1$

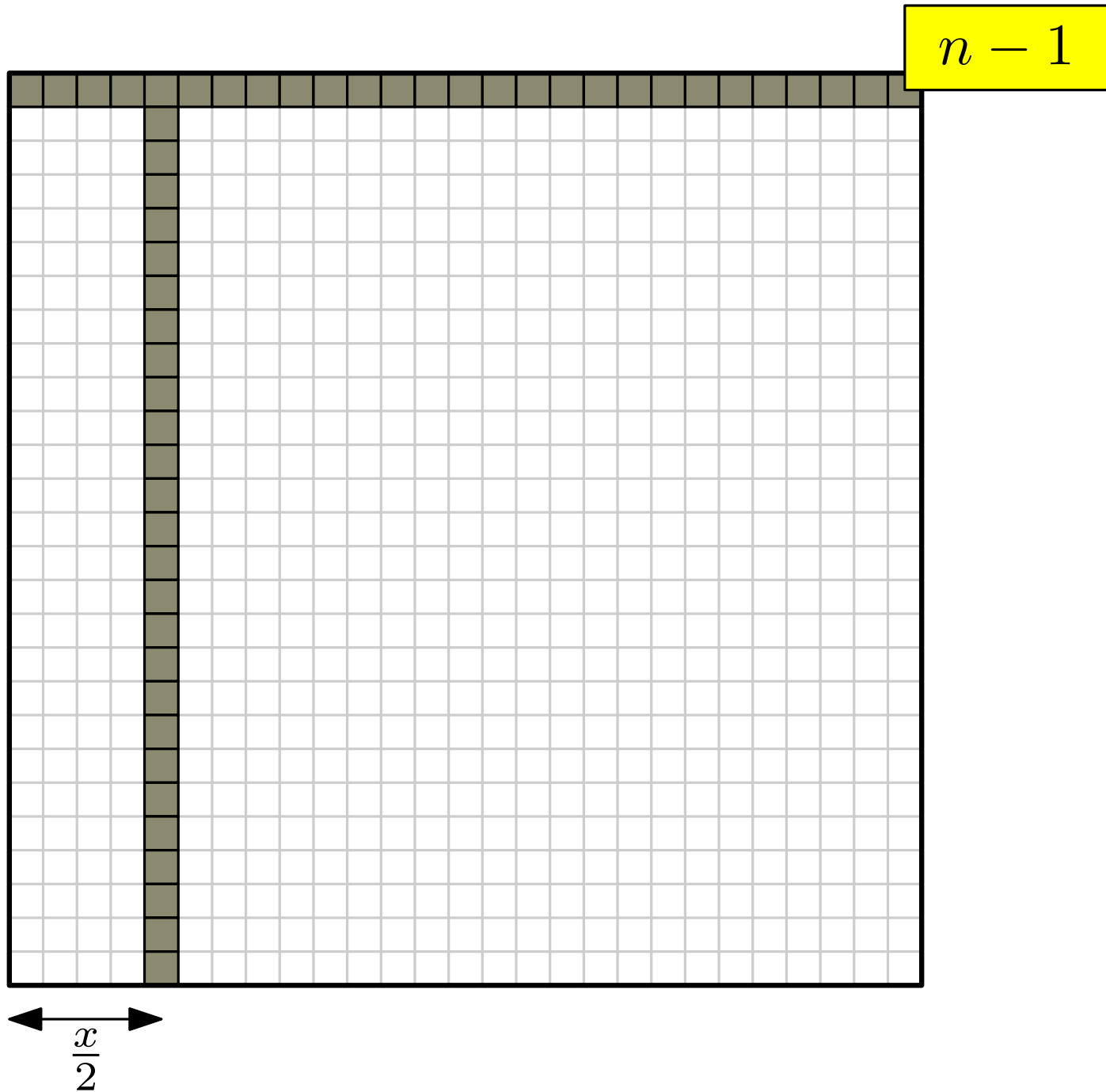


Upper Bound

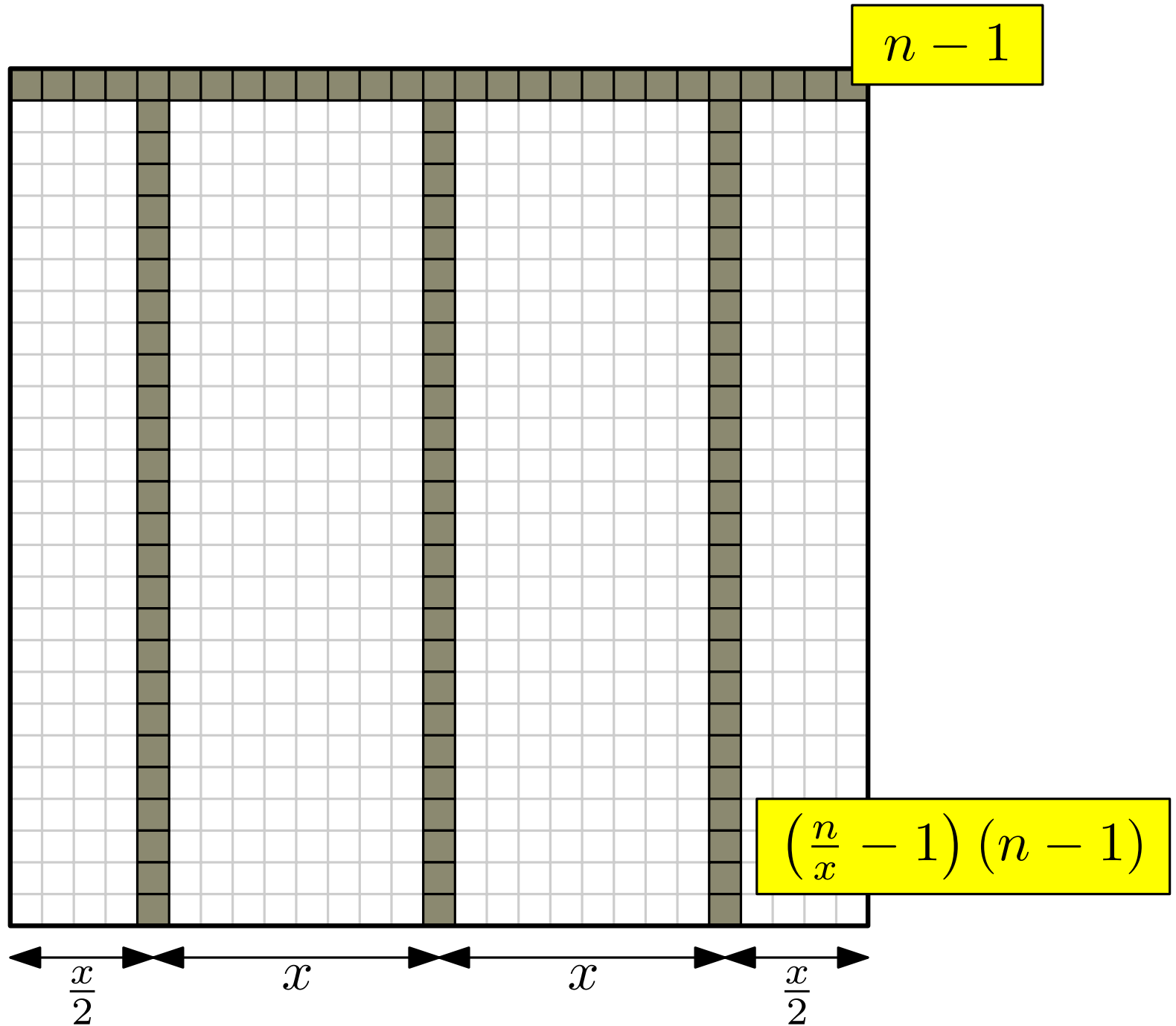
$n - 1$



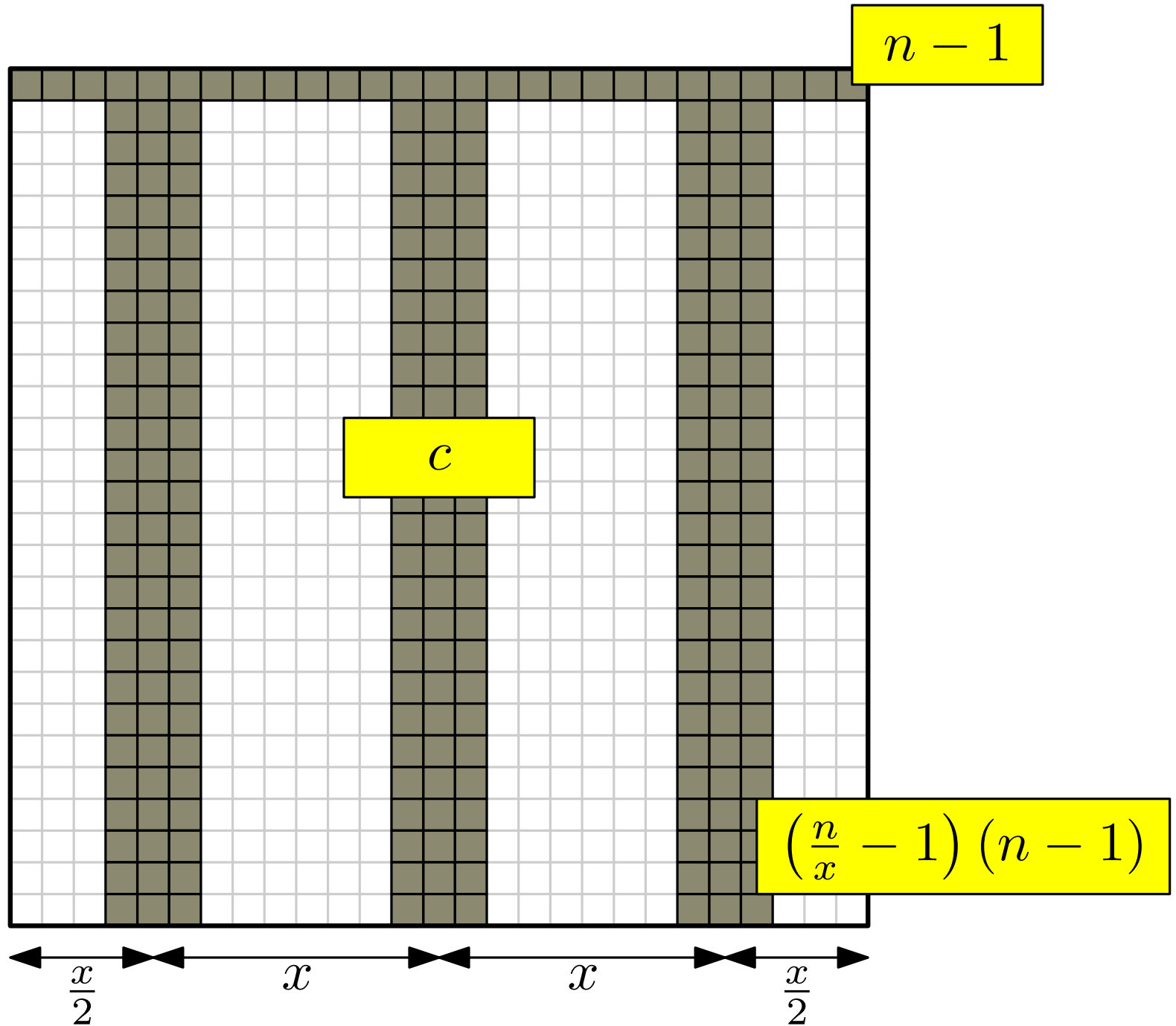
Upper Bound



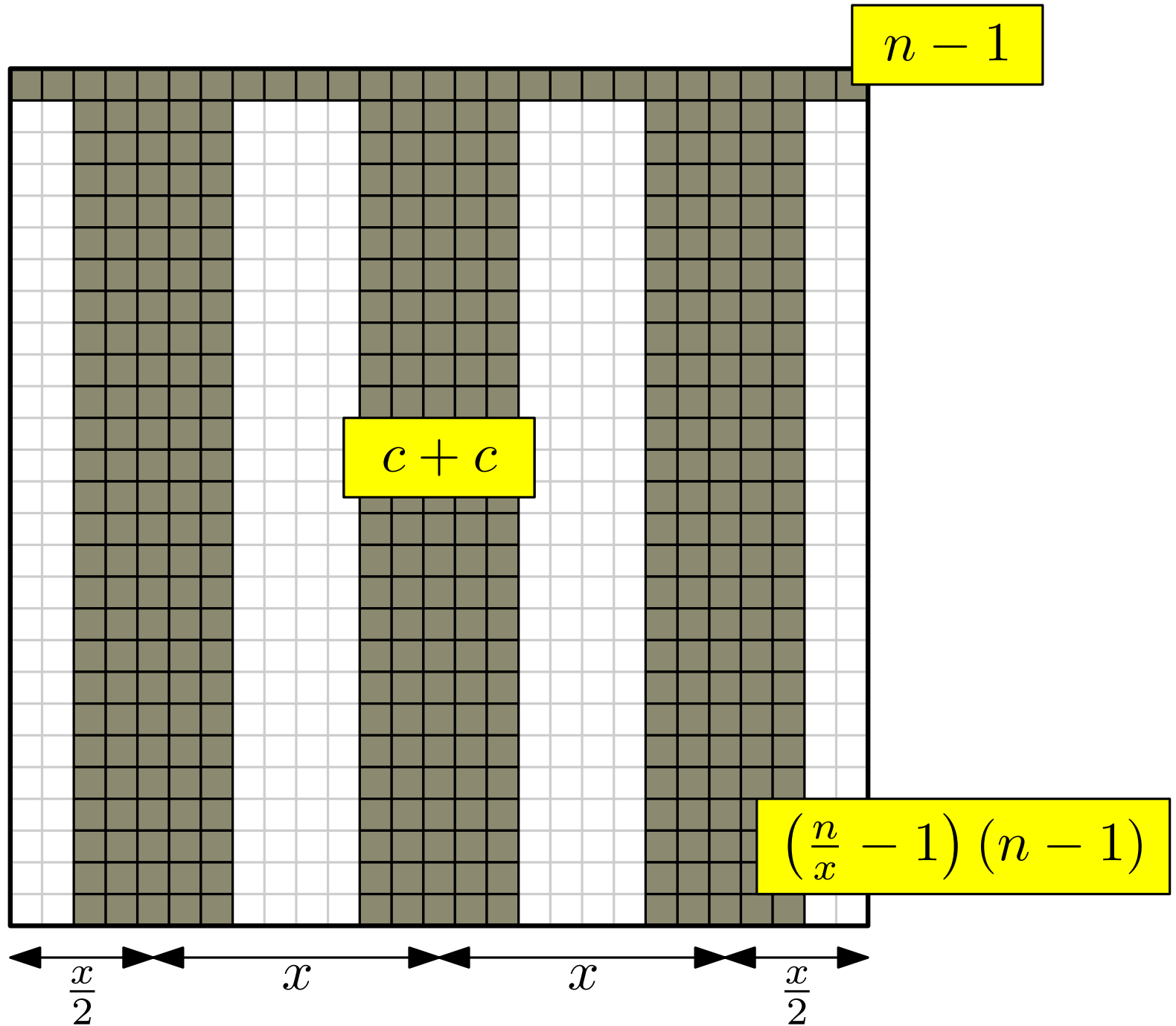
Upper Bound



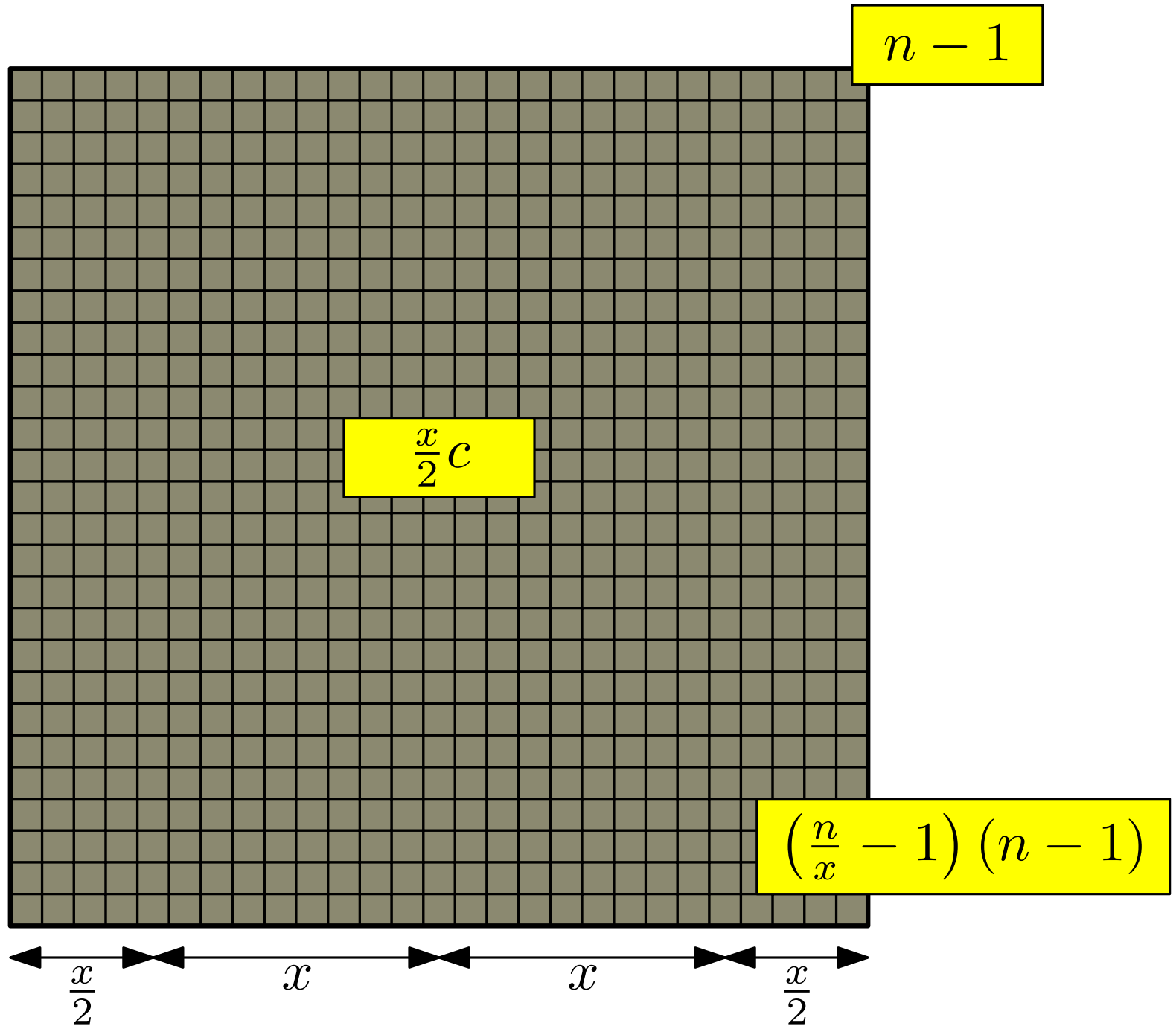
Upper Bound



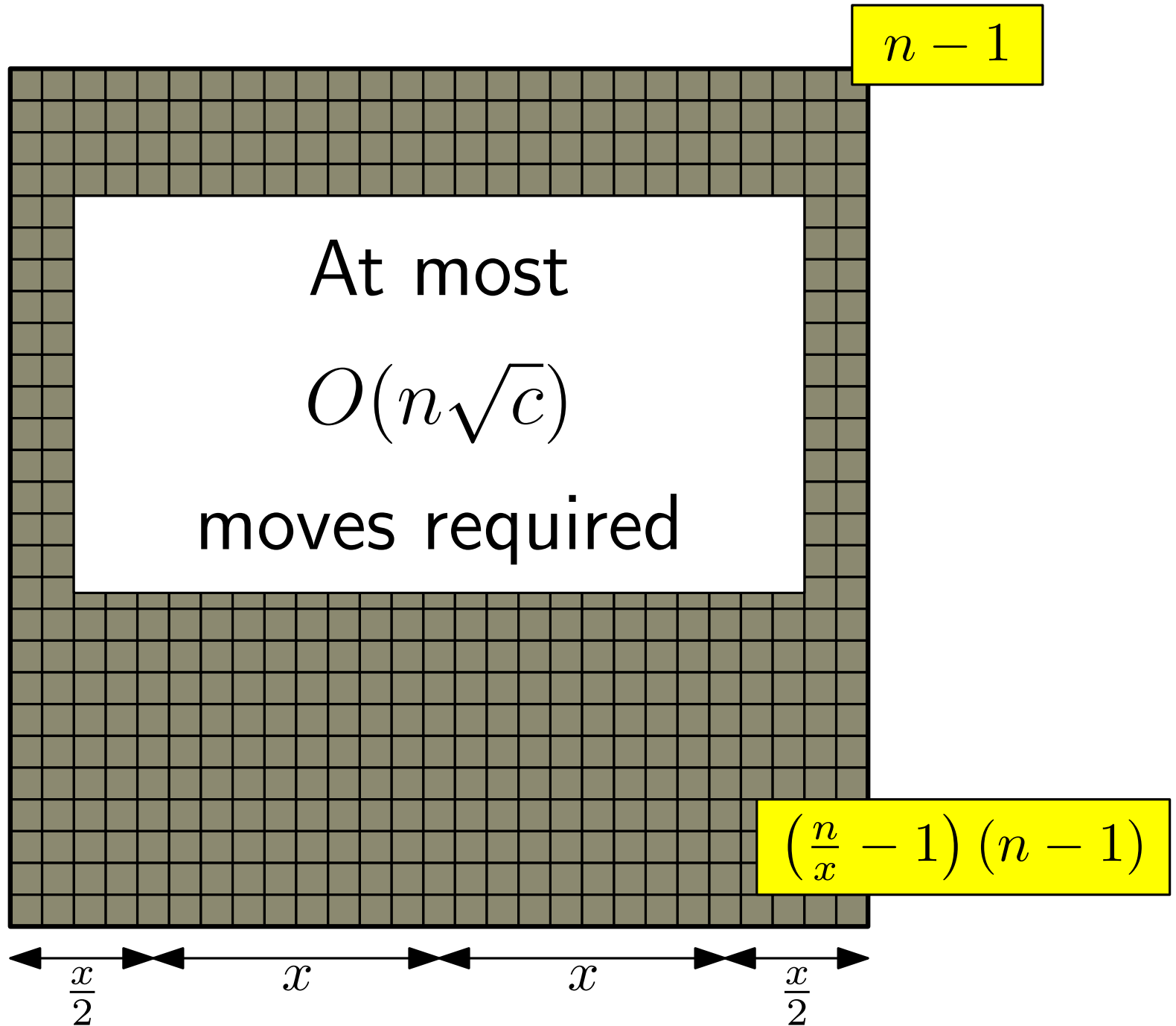
Upper Bound



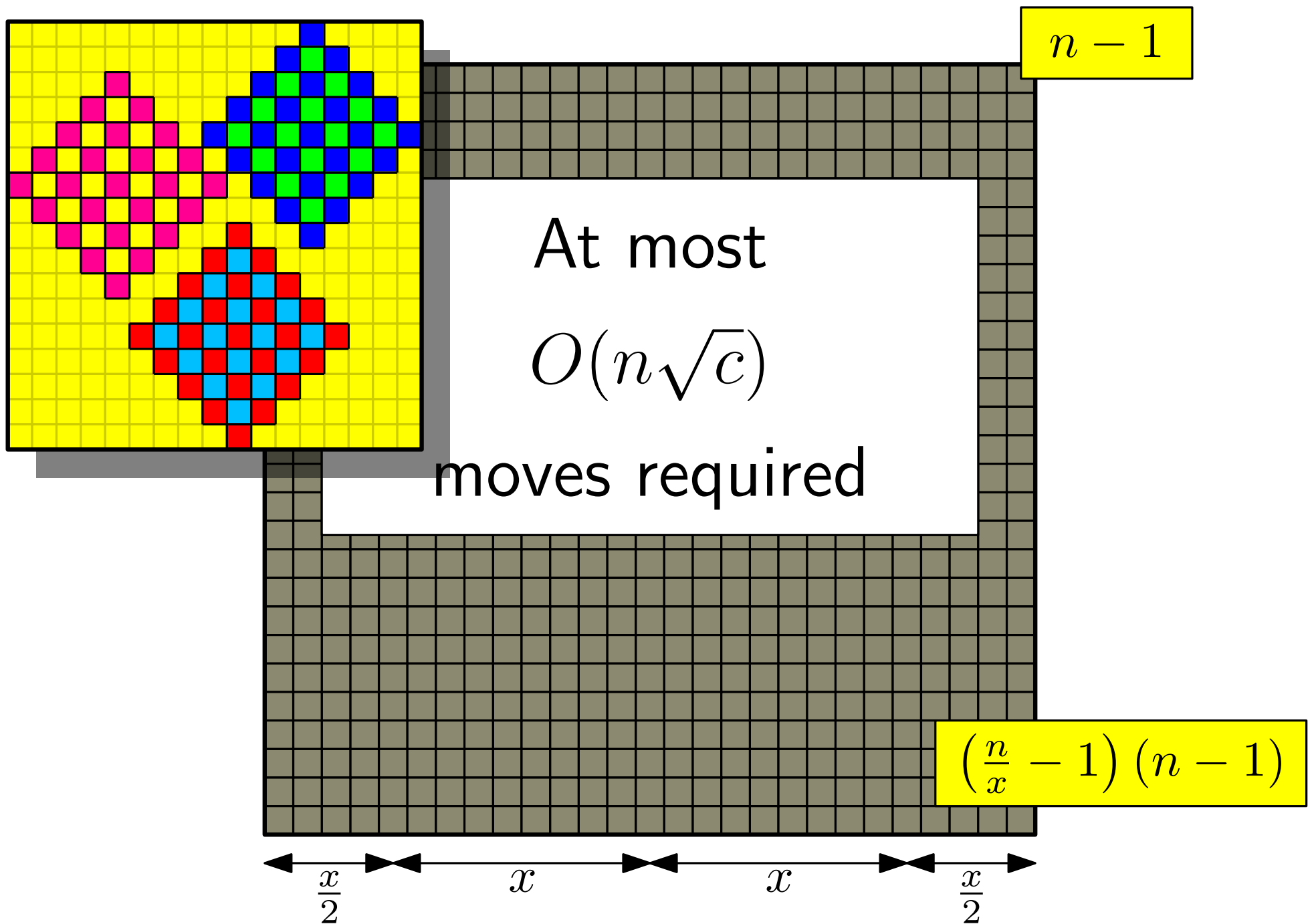
Upper Bound



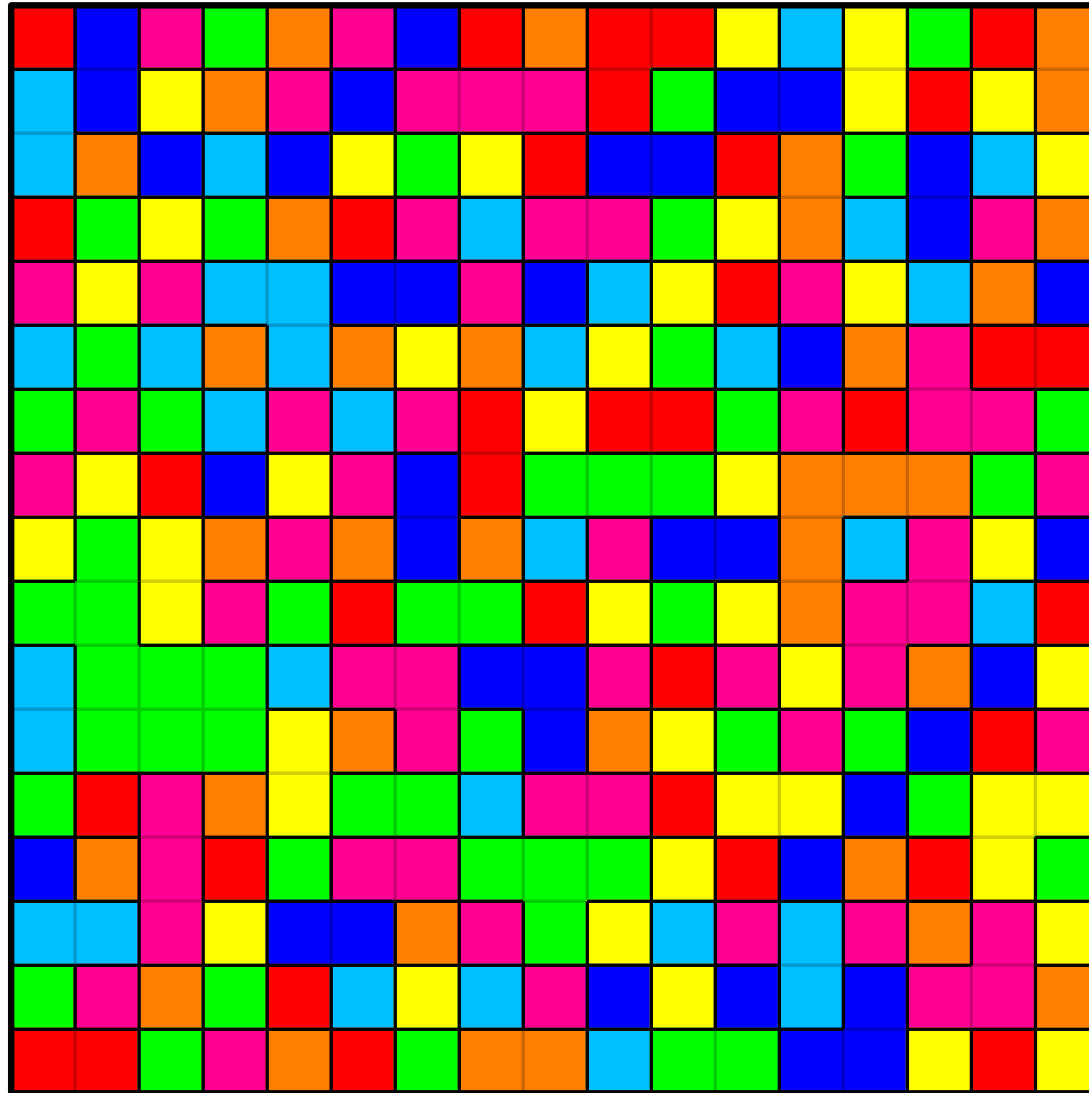
Upper Bound



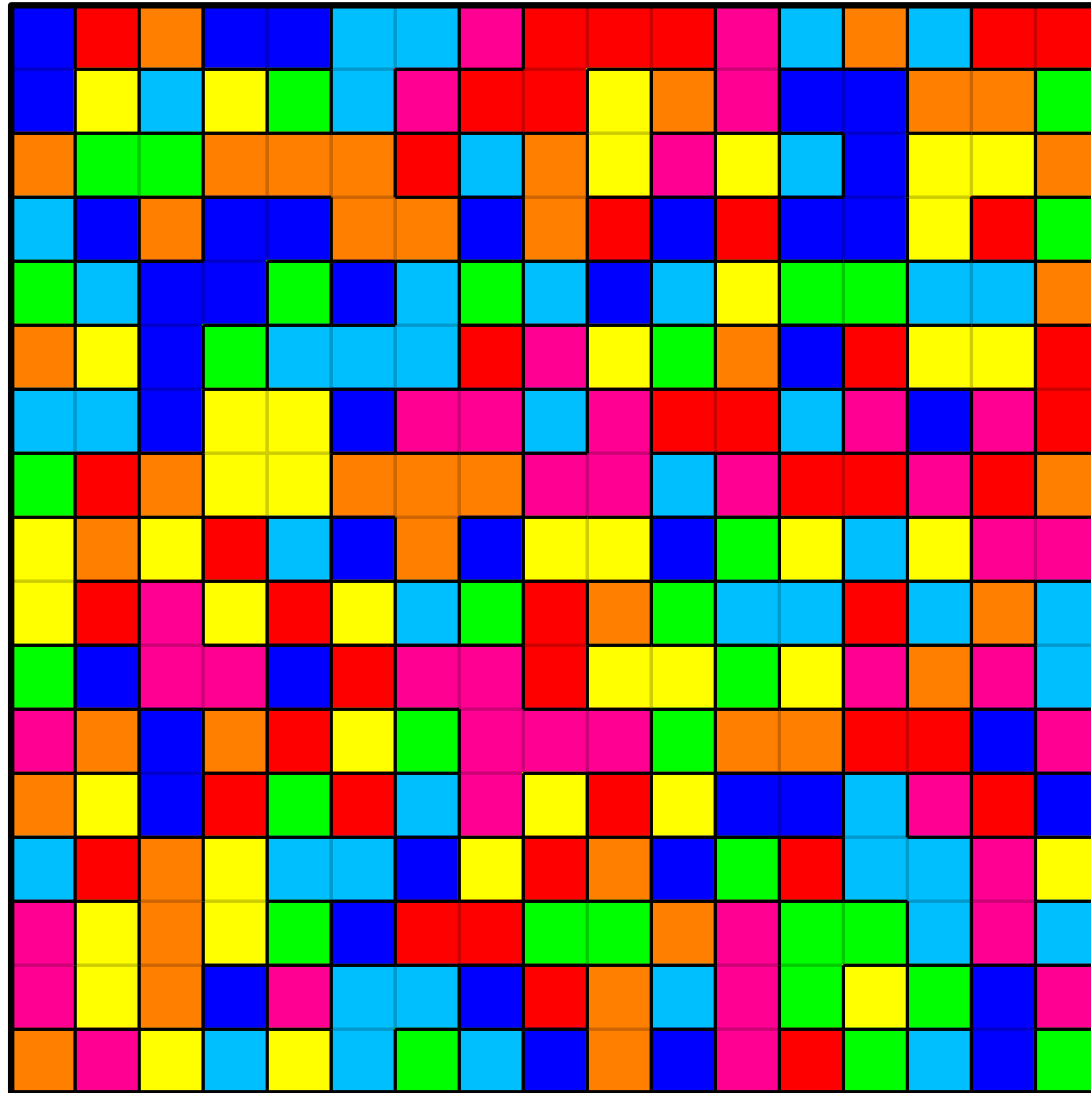
Upper Bound



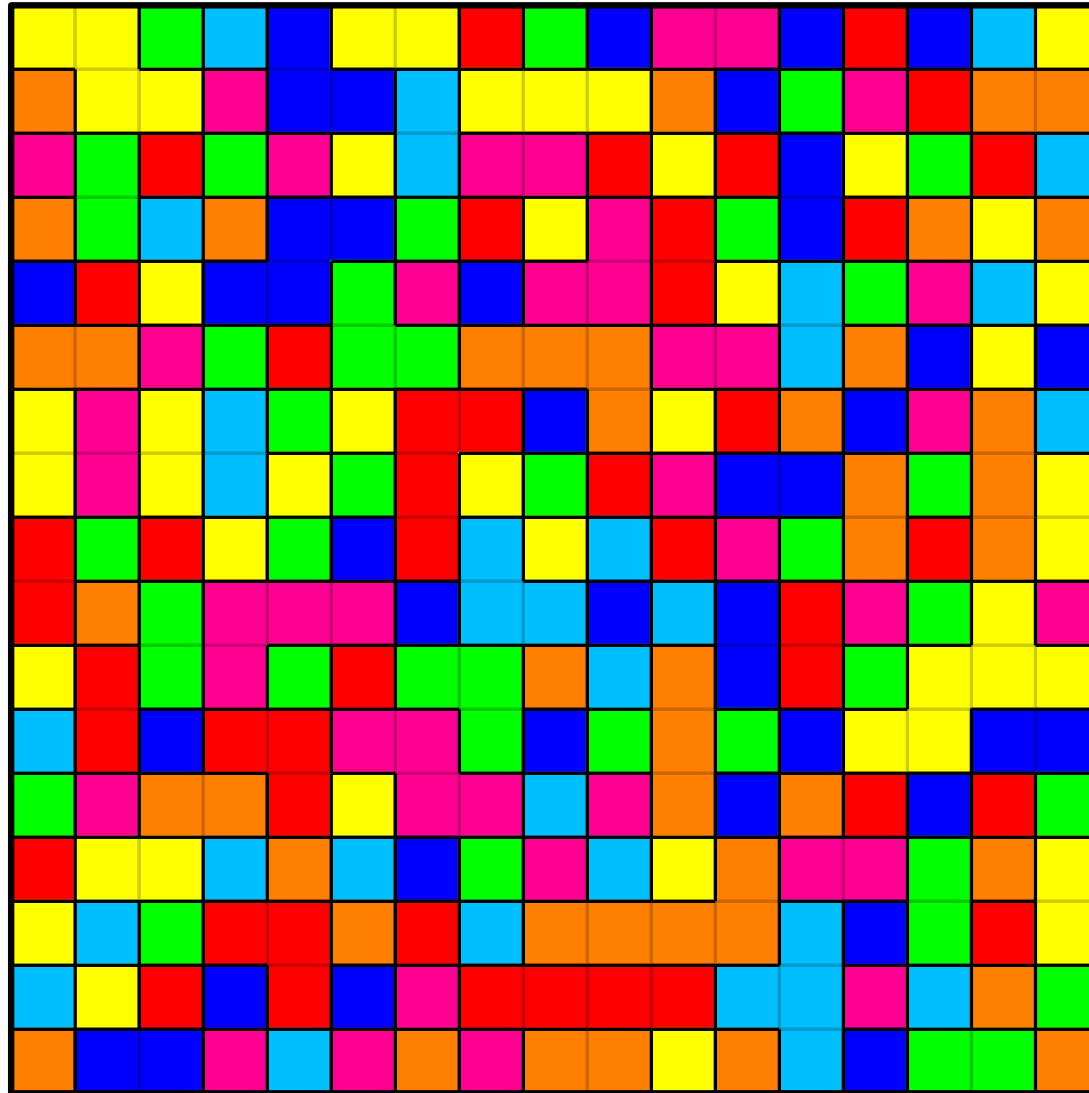
Random Boards



Random Boards

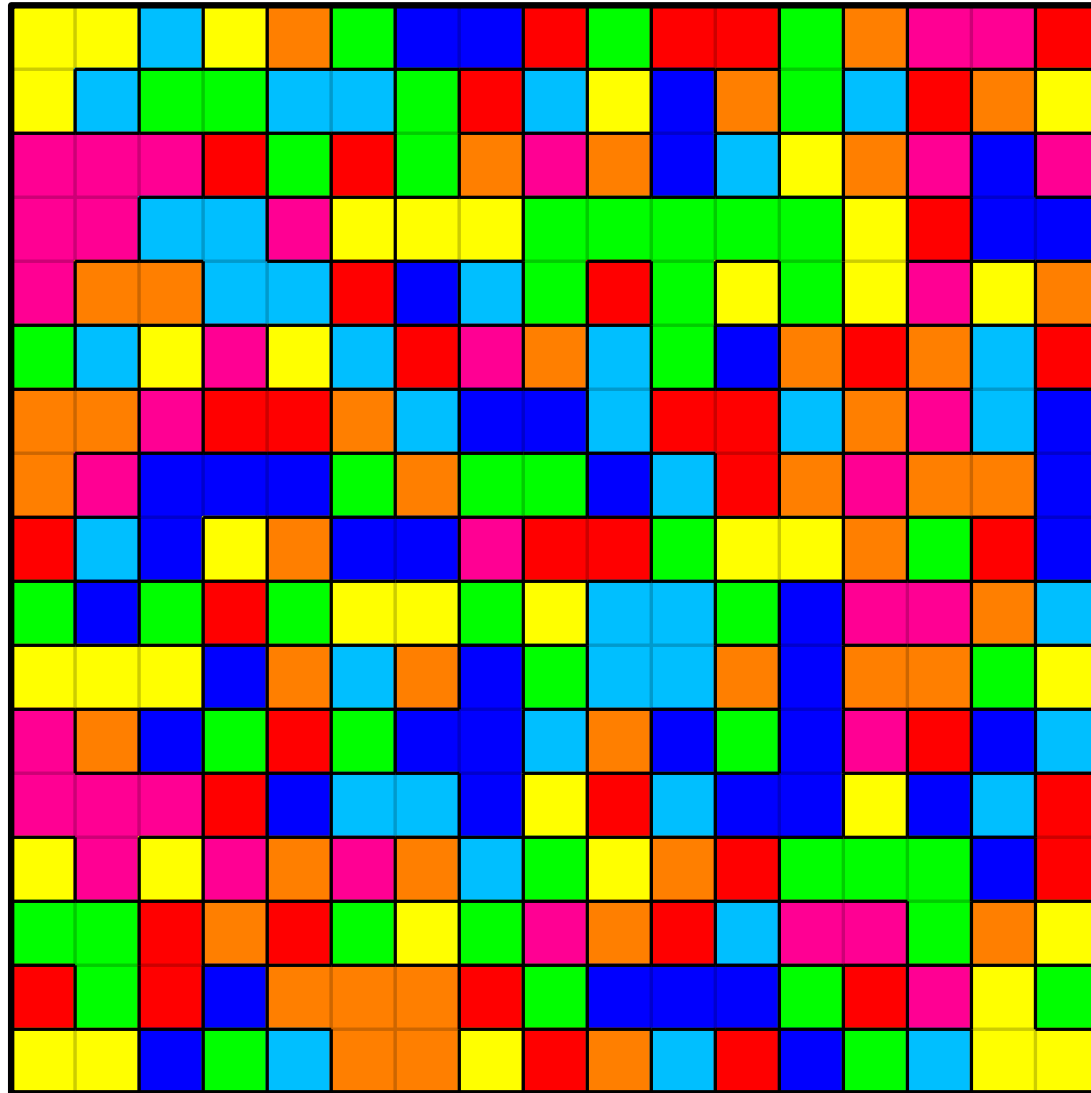


Random Boards



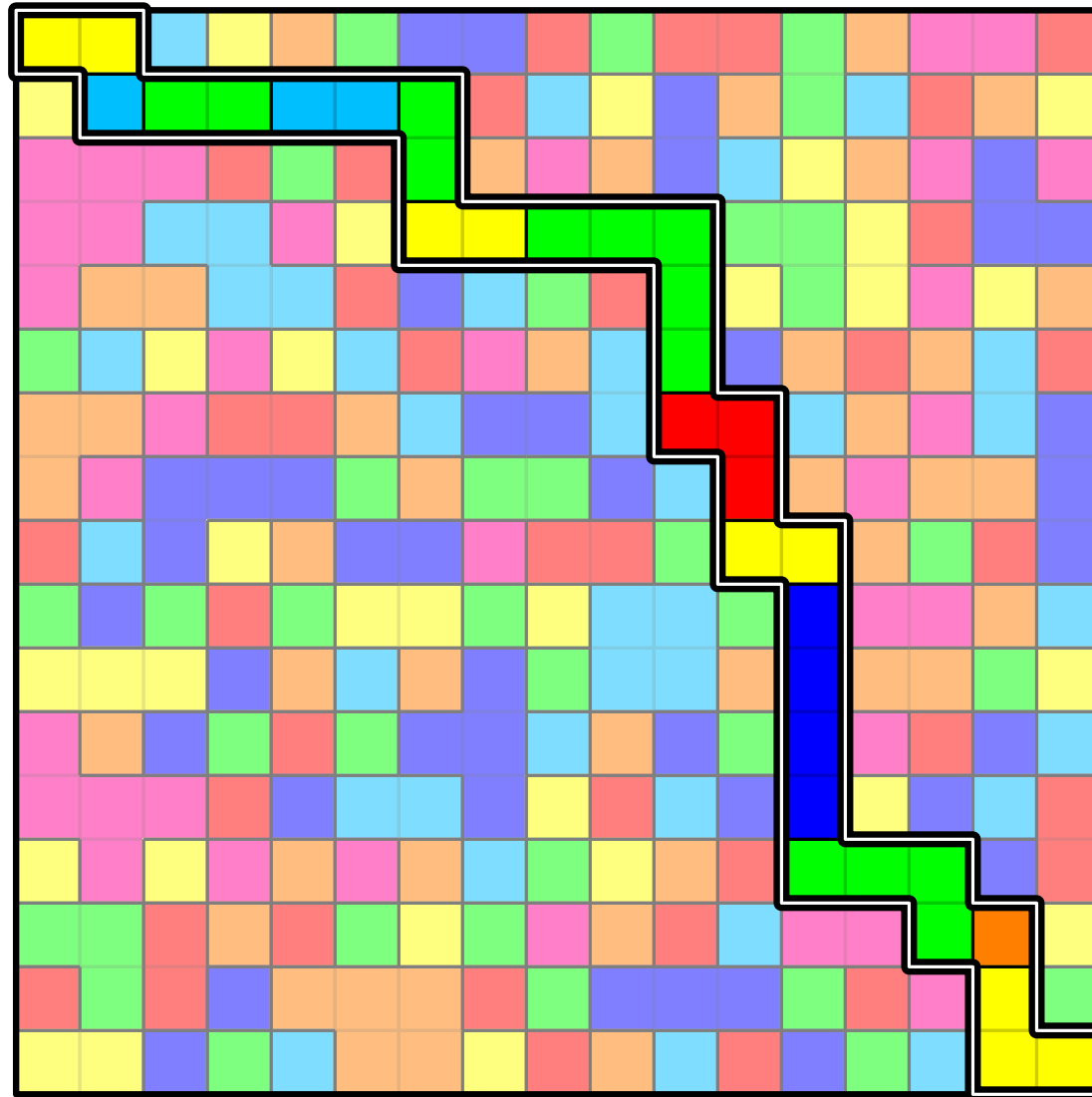
Random Boards

m moves to flood the board



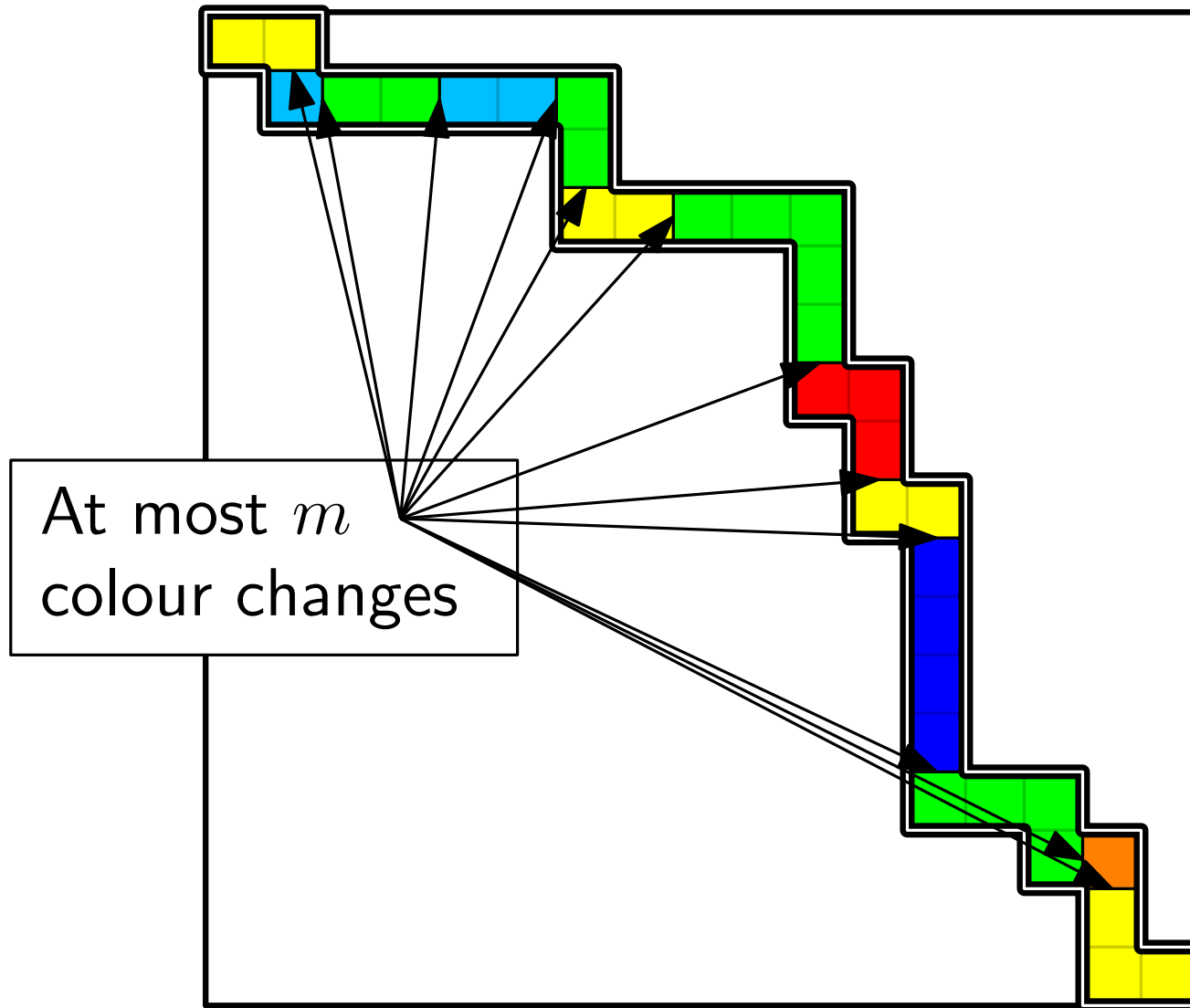
Random Boards

m moves to flood the board



Random Boards

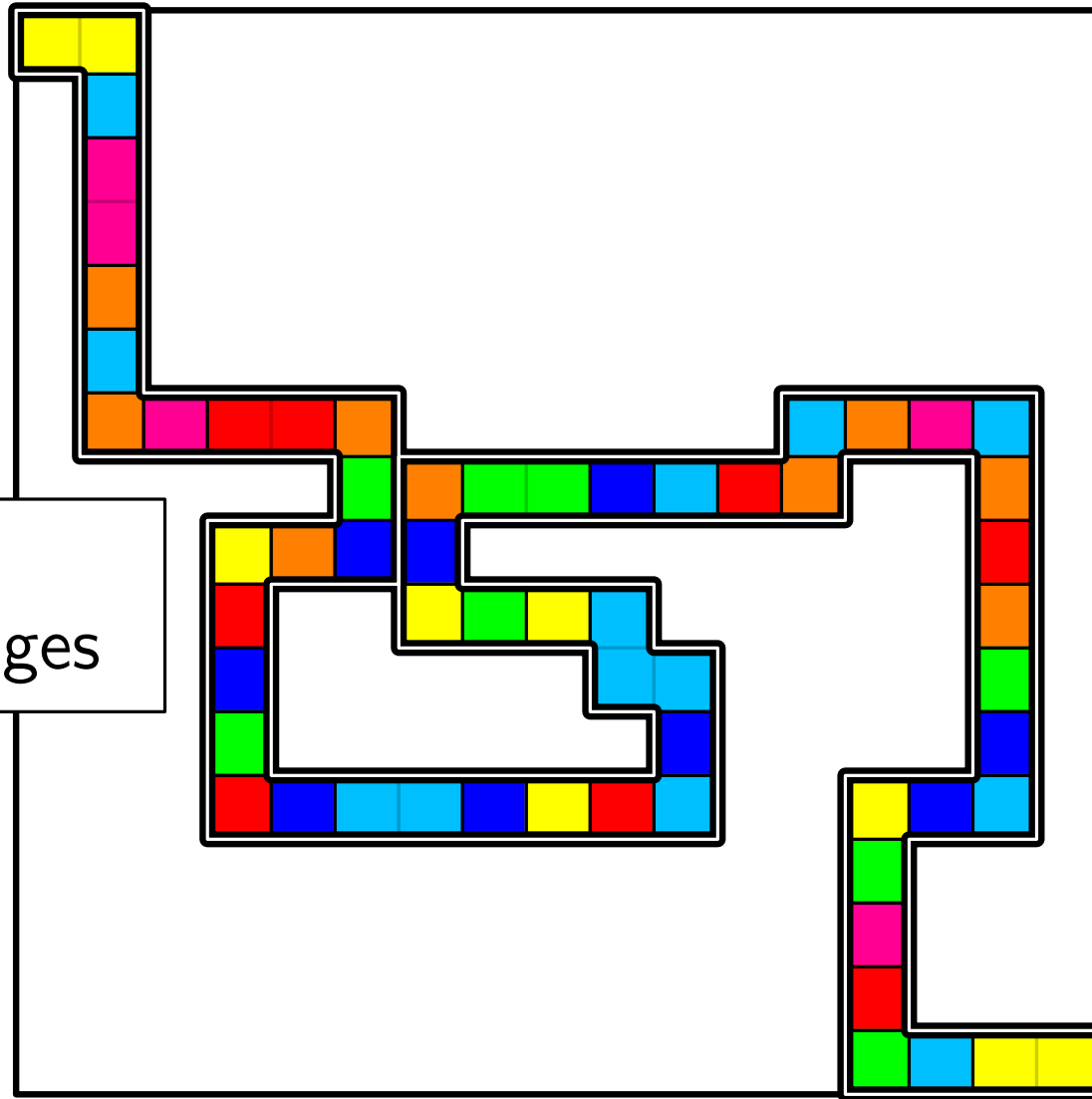
m moves to flood the board



Random Boards

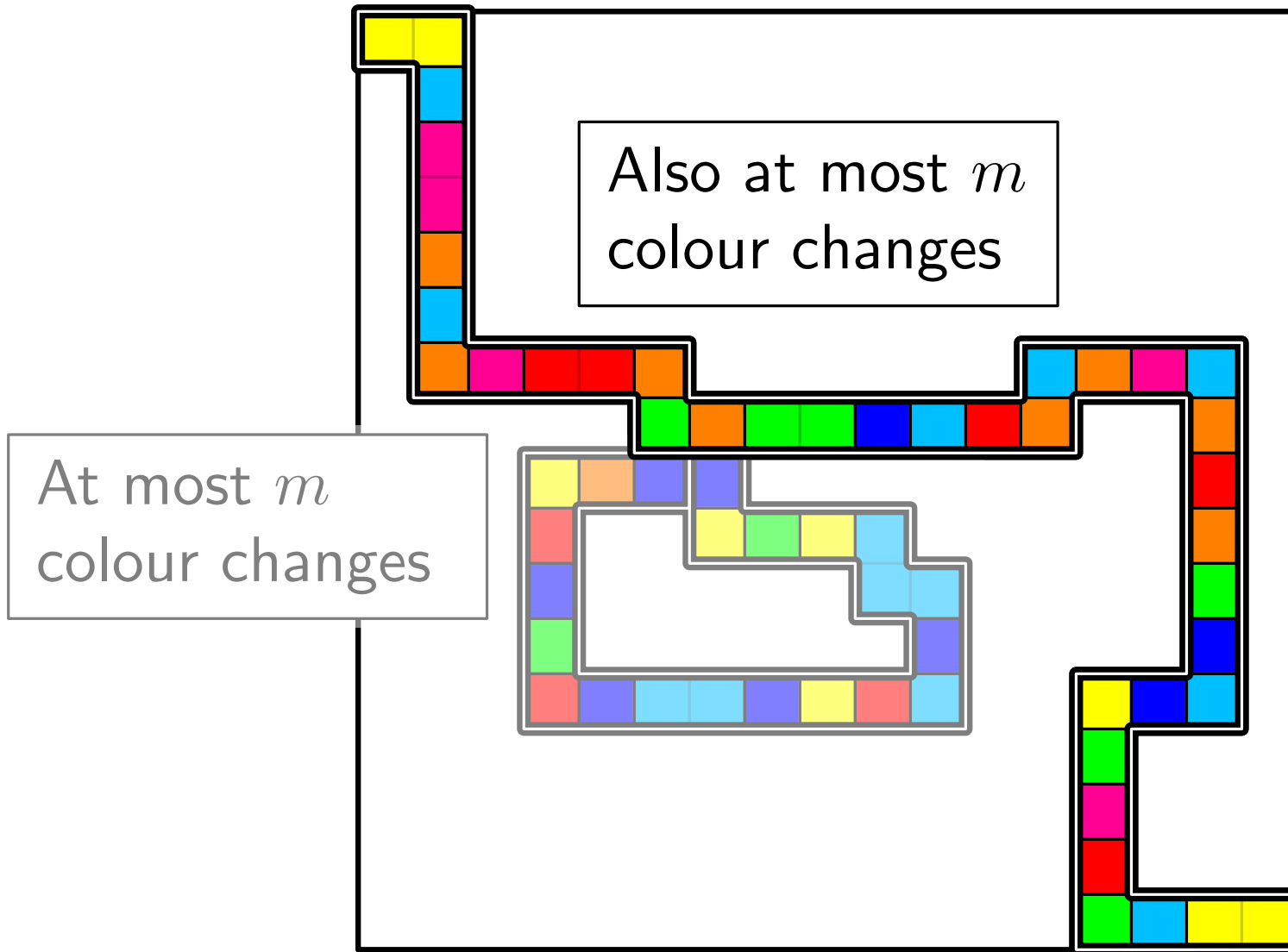
m moves to flood the board

At most m
colour changes



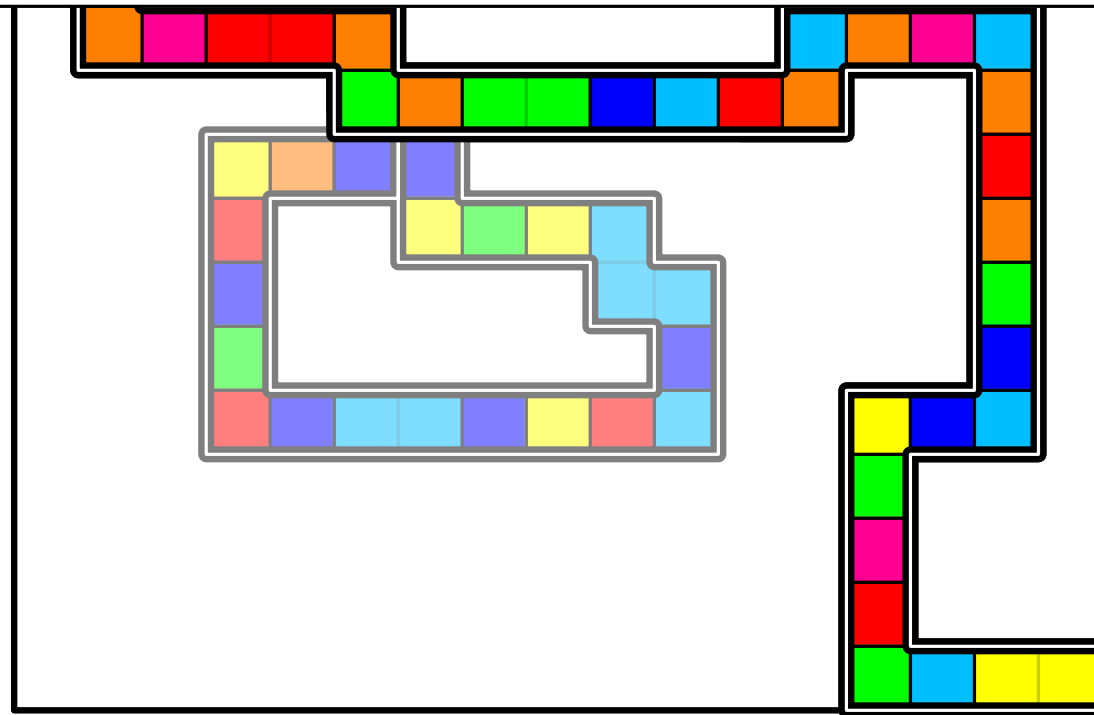
Random Boards

m moves to flood the board



Random Boards

We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most k colours changes. The bound depends on k , number of colours c and the length of the path.



Random Boards

We derive an upper bound on the probability that an arbitrary non-touching path from the top left to the bottom right tile has at most k colours changes. The bound depends on k , number of colours c and the length of the path.



The probability that there exists *any* non-touching path from the top left to the bottom right tile with at most k colour changes is upper bounded by the union bound over *all* non-touching paths from the top left to the bottom right tile.



Random Boards

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k is order n

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This union bound is an upper bound on the probability that the board is flooded within k moves.

Random Boards

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Probability less than $e^{-\Omega(n)}$
(for 3 or more colours)

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Random Boards

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Conclusion

The number of moves required to flood a random board is $\Omega(n)$ with high probability.

the bottom right tile.

Probability less than $e^{-\Omega(n)}$
(for 3 or more colours)

This union bound is an upper bound on the probability that the board is flooded within k moves.

Thank You!

Don't forget our website:

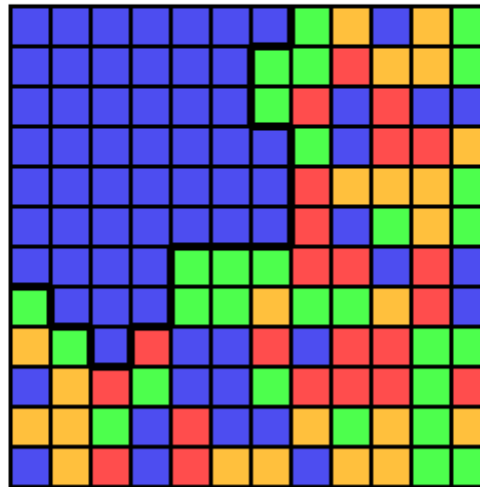
<http://floodit.cs.bris.ac.uk/>

Play Replays [help](#) | [paper](#)

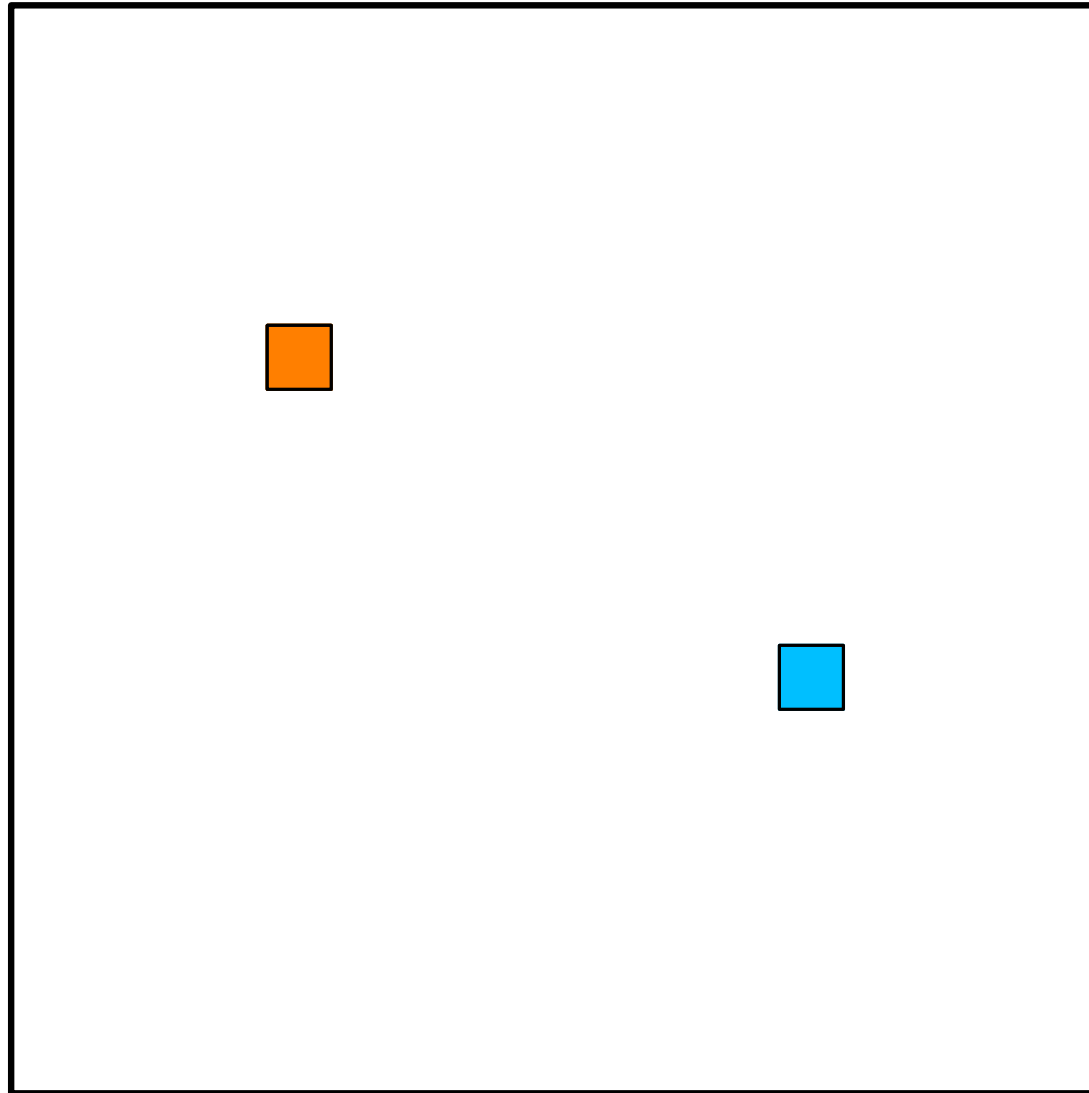
Size: 12x12 ▾ Colours: 4 ▾ Board: 1 ▾ Play

Special Boards: [3 Colour NPC](#) | [4 Colour NPC](#) ([what are these?](#))

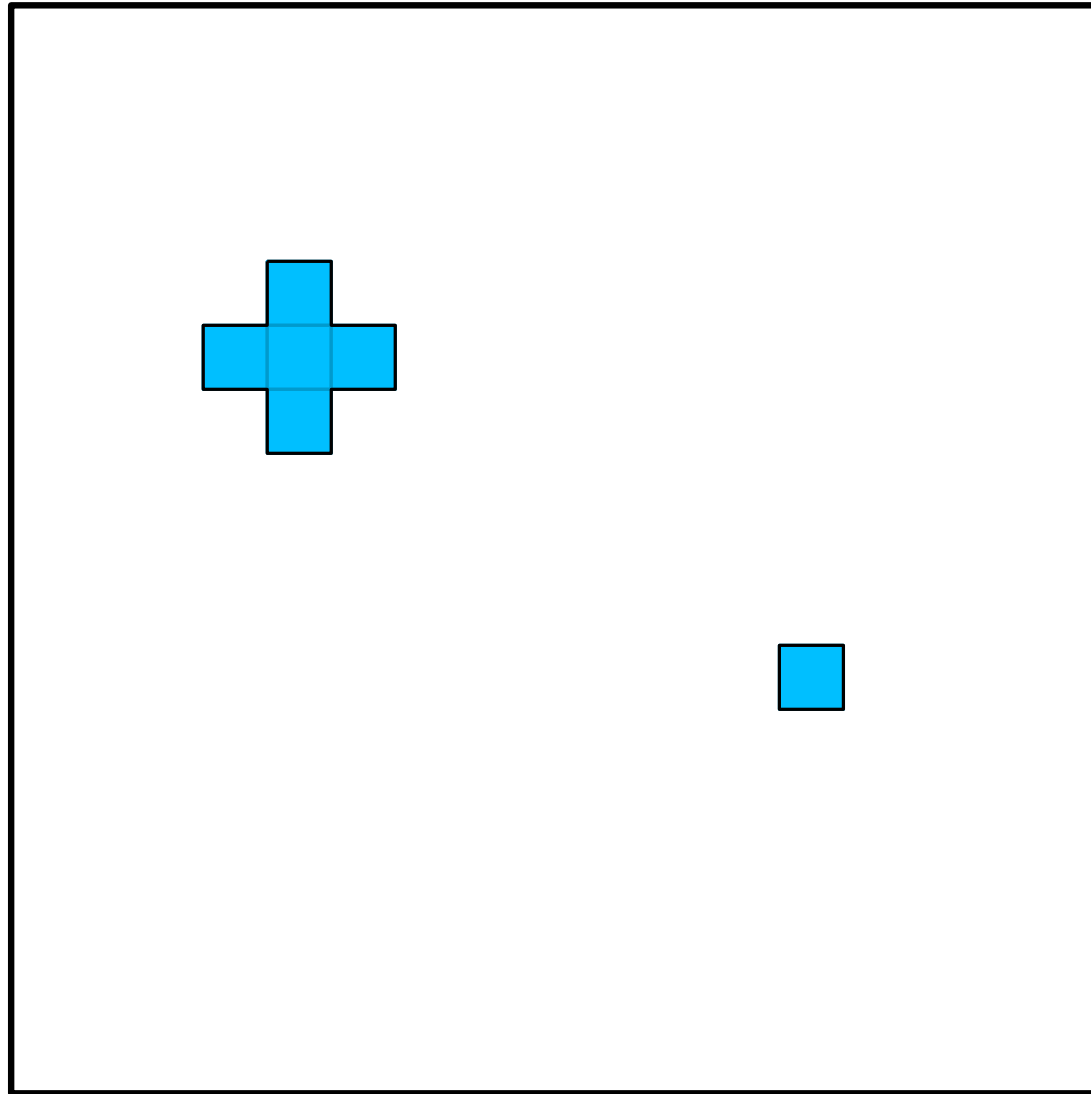
Move: 10 [Par: 18](#)



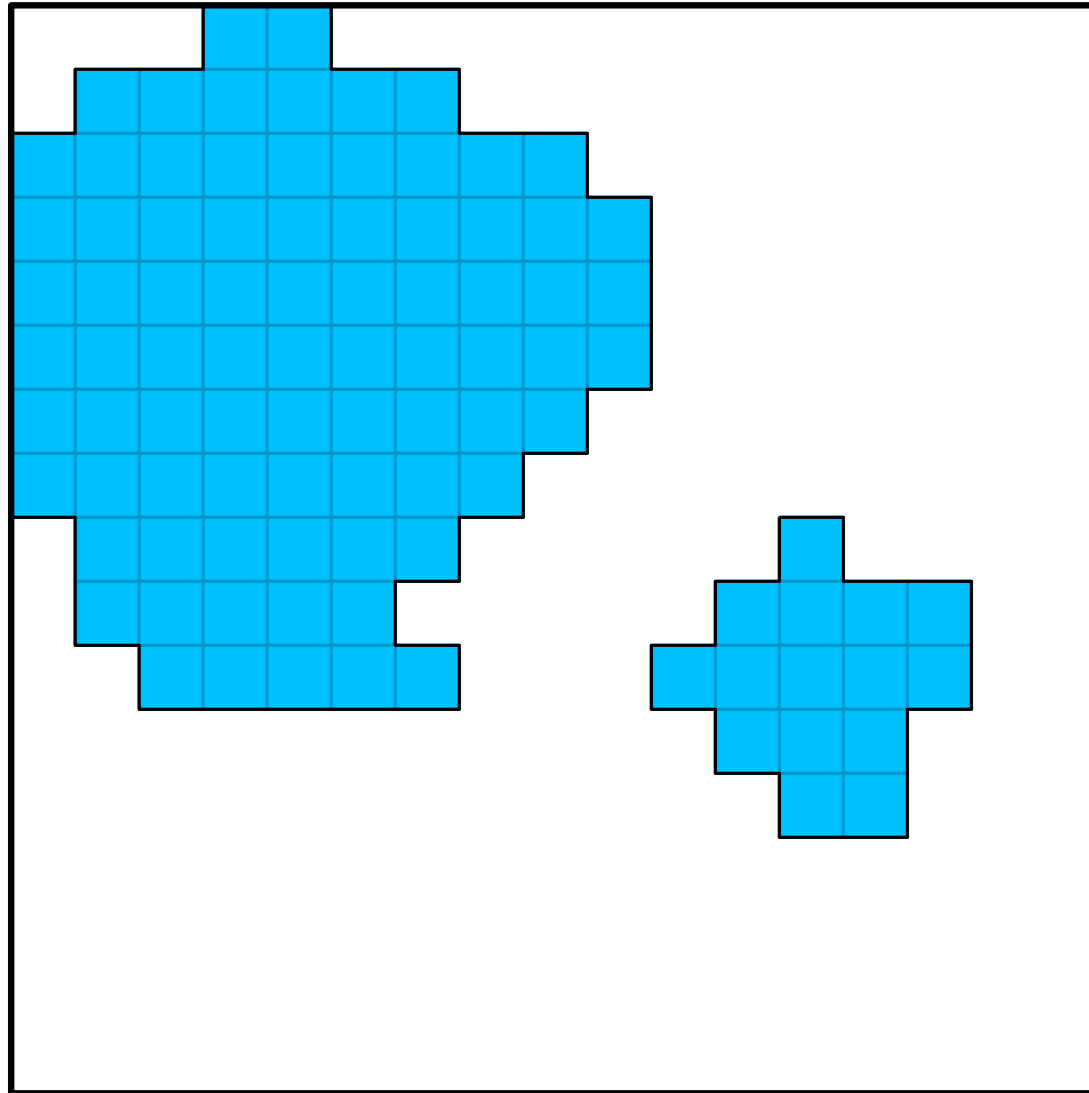
2 Colours (Free)



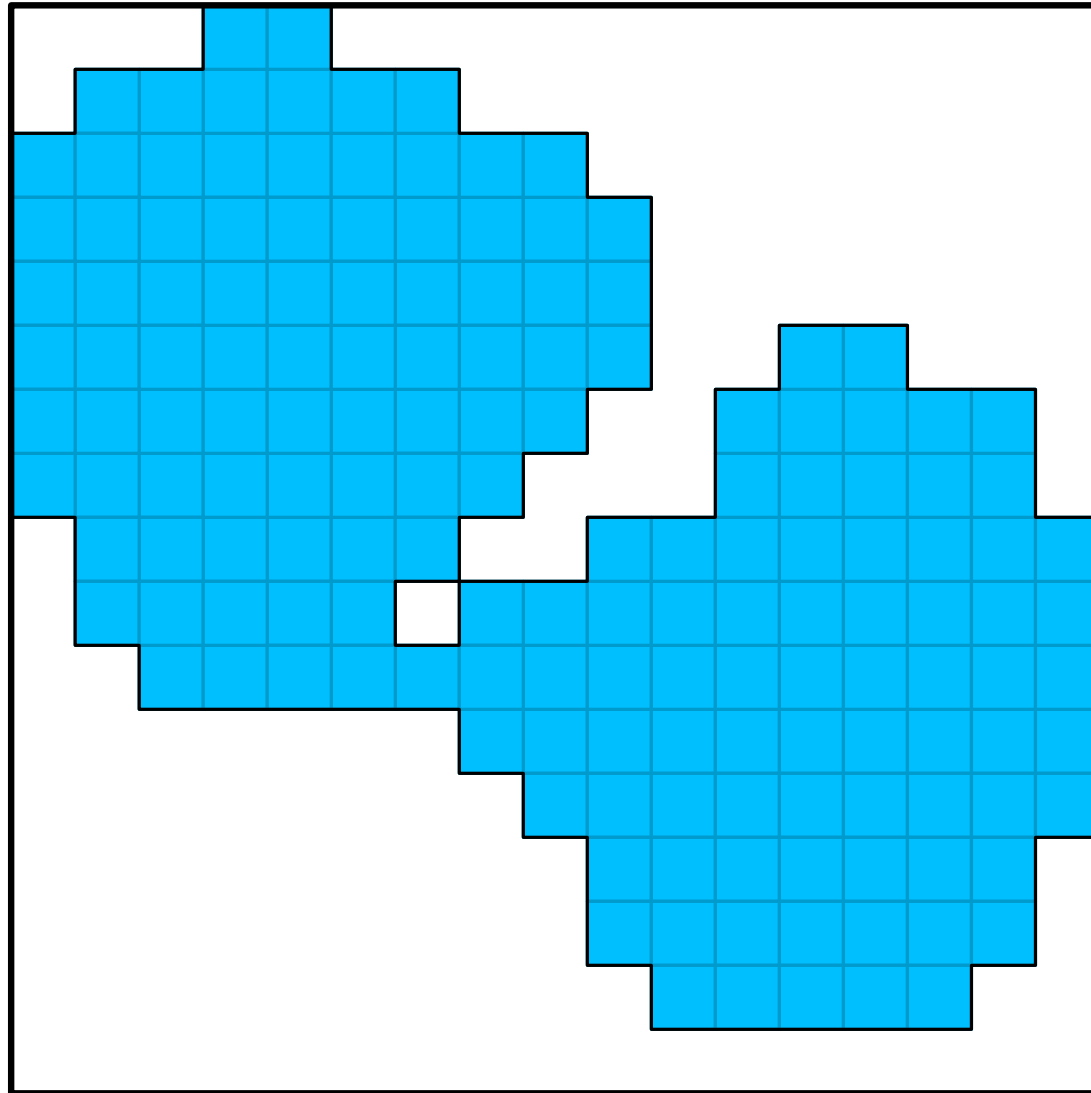
2 Colours (Free)



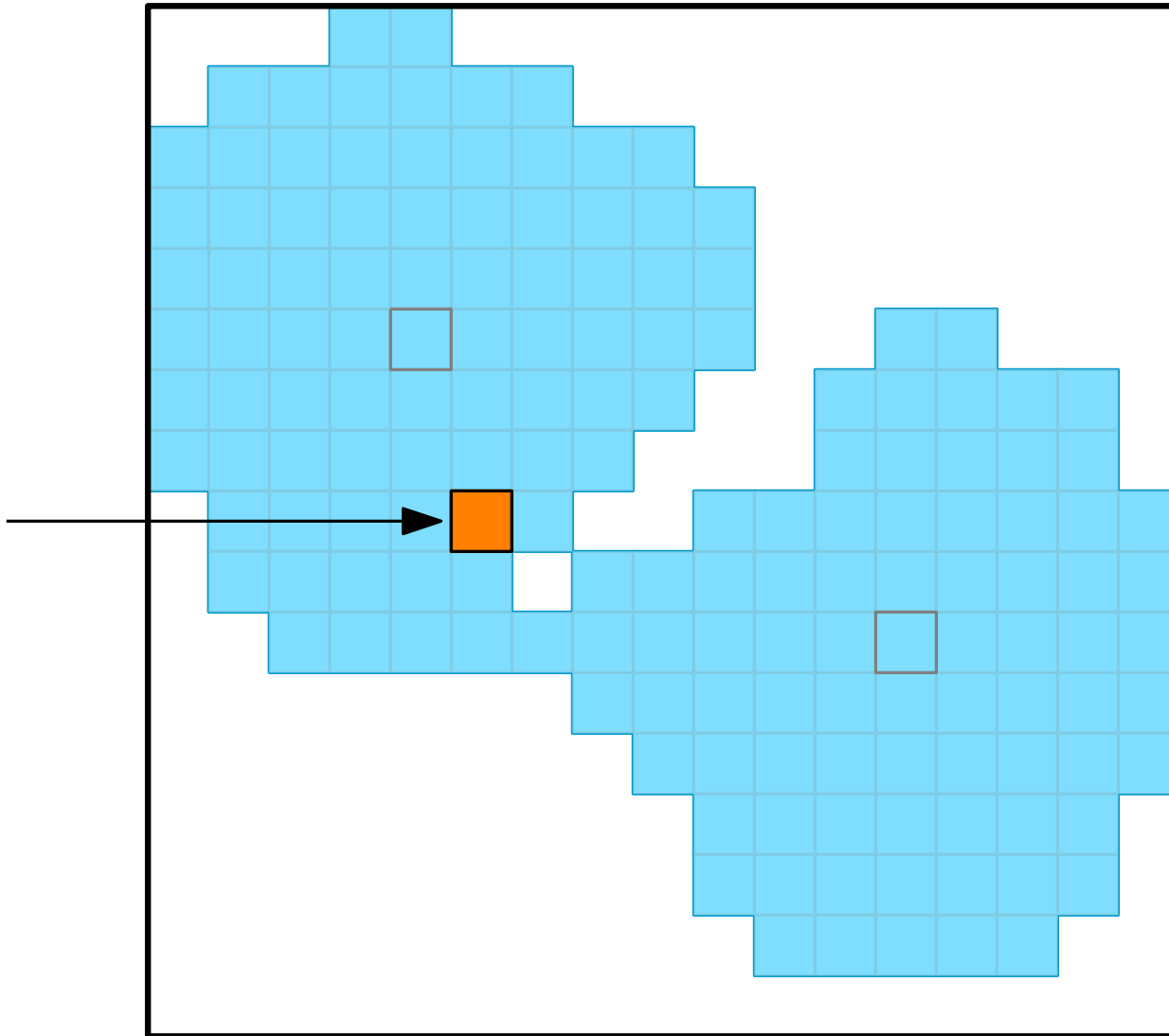
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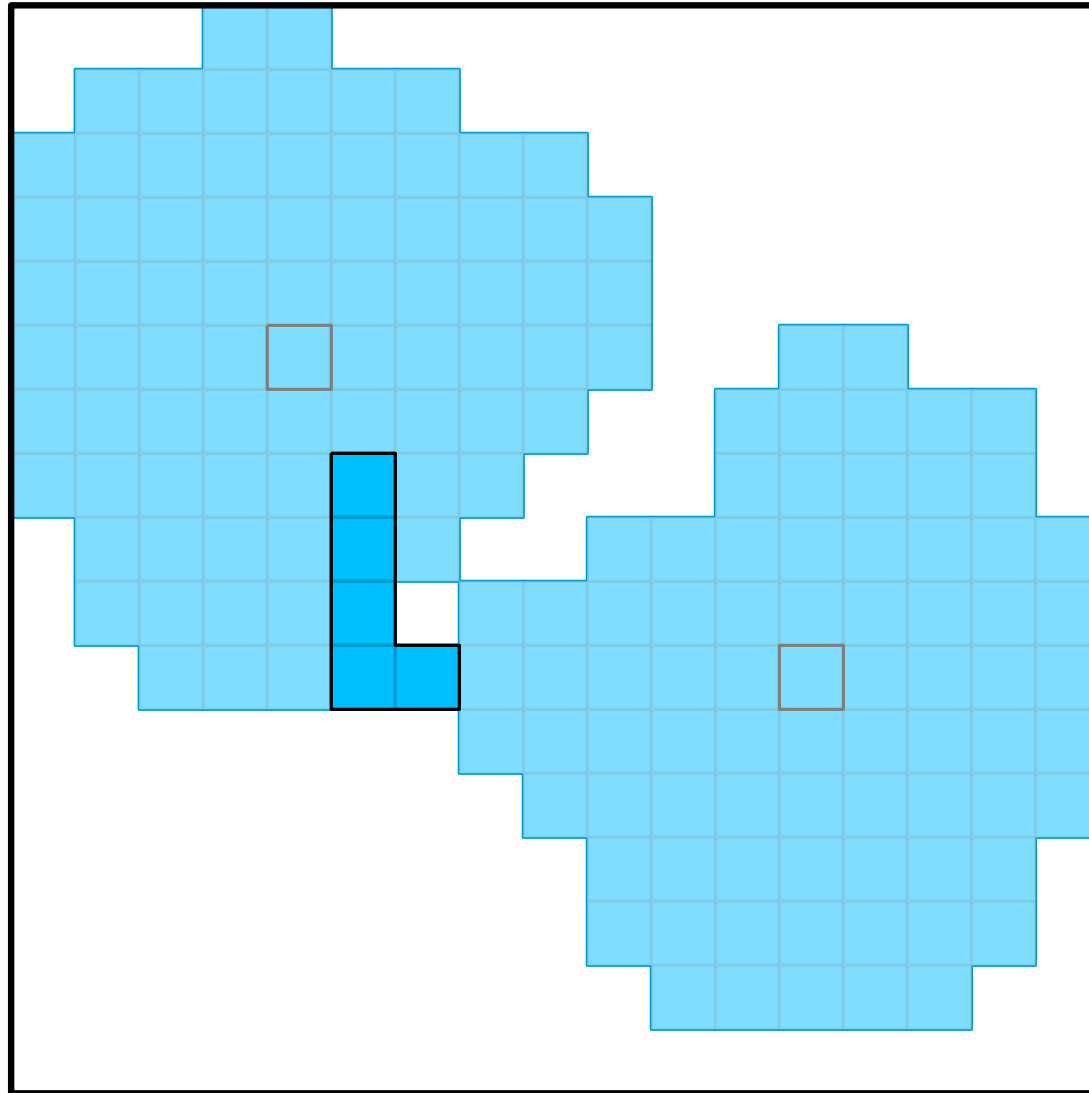
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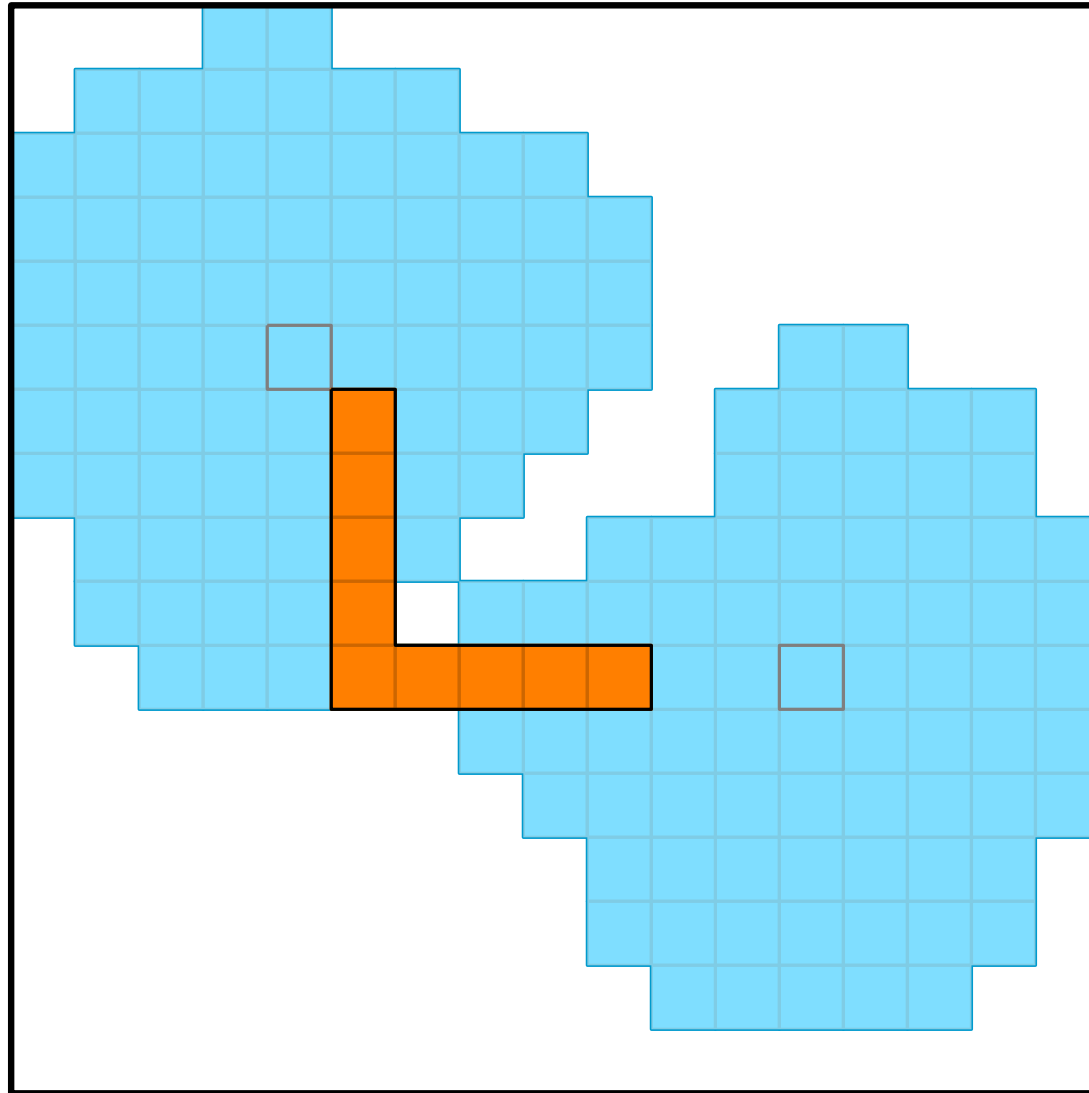
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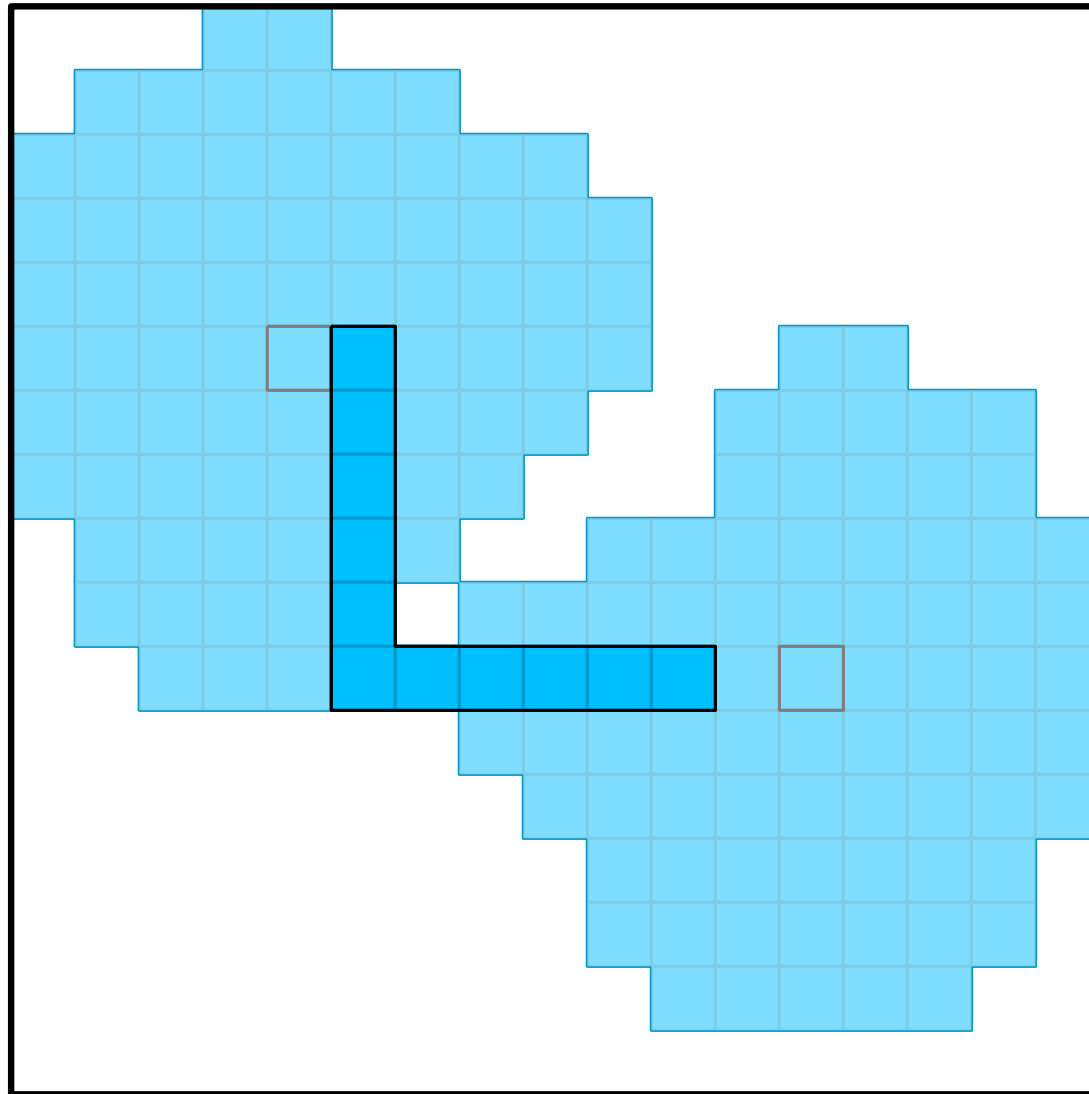
2 Colours (Free)



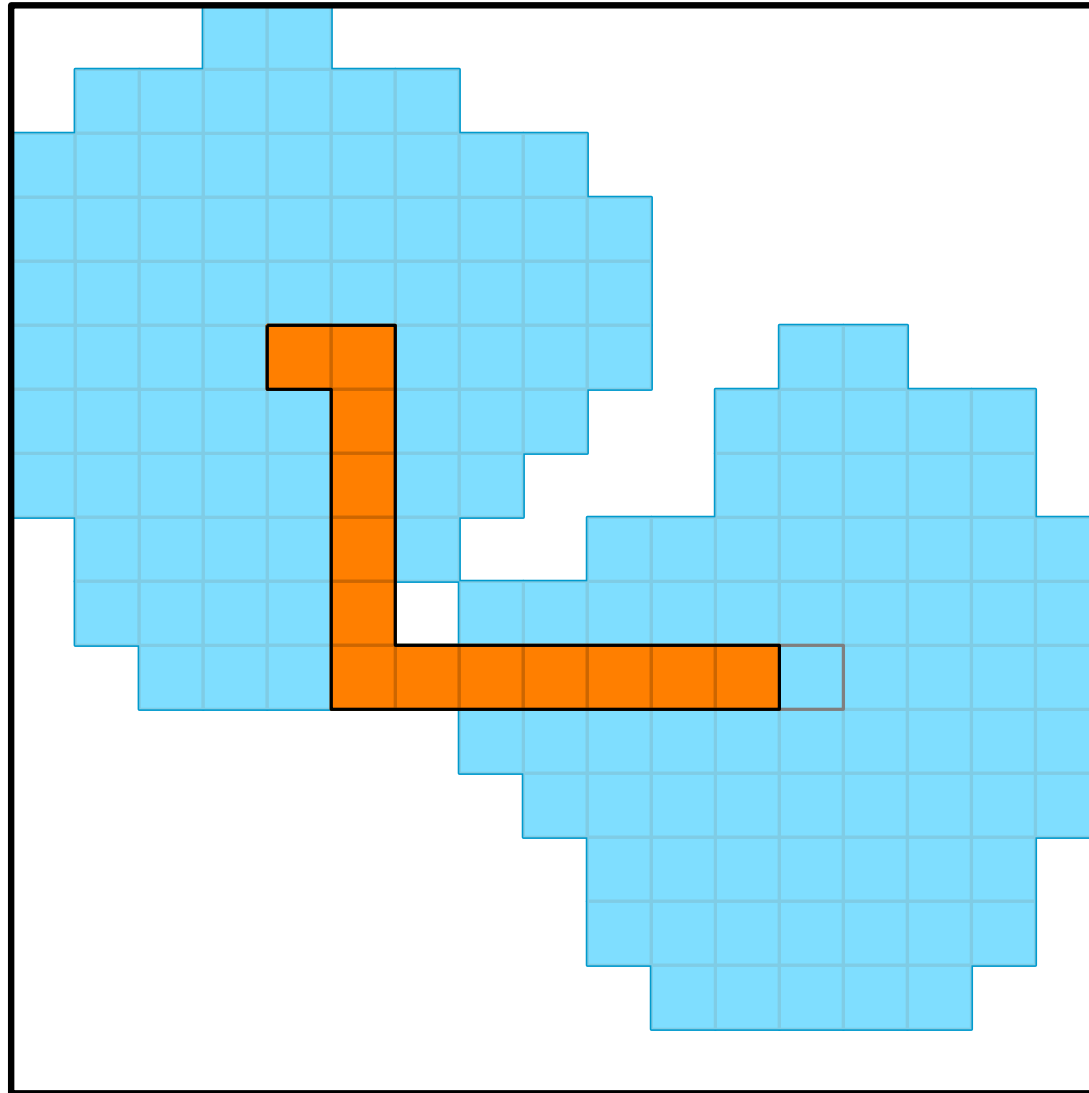
2 Colours (Free)



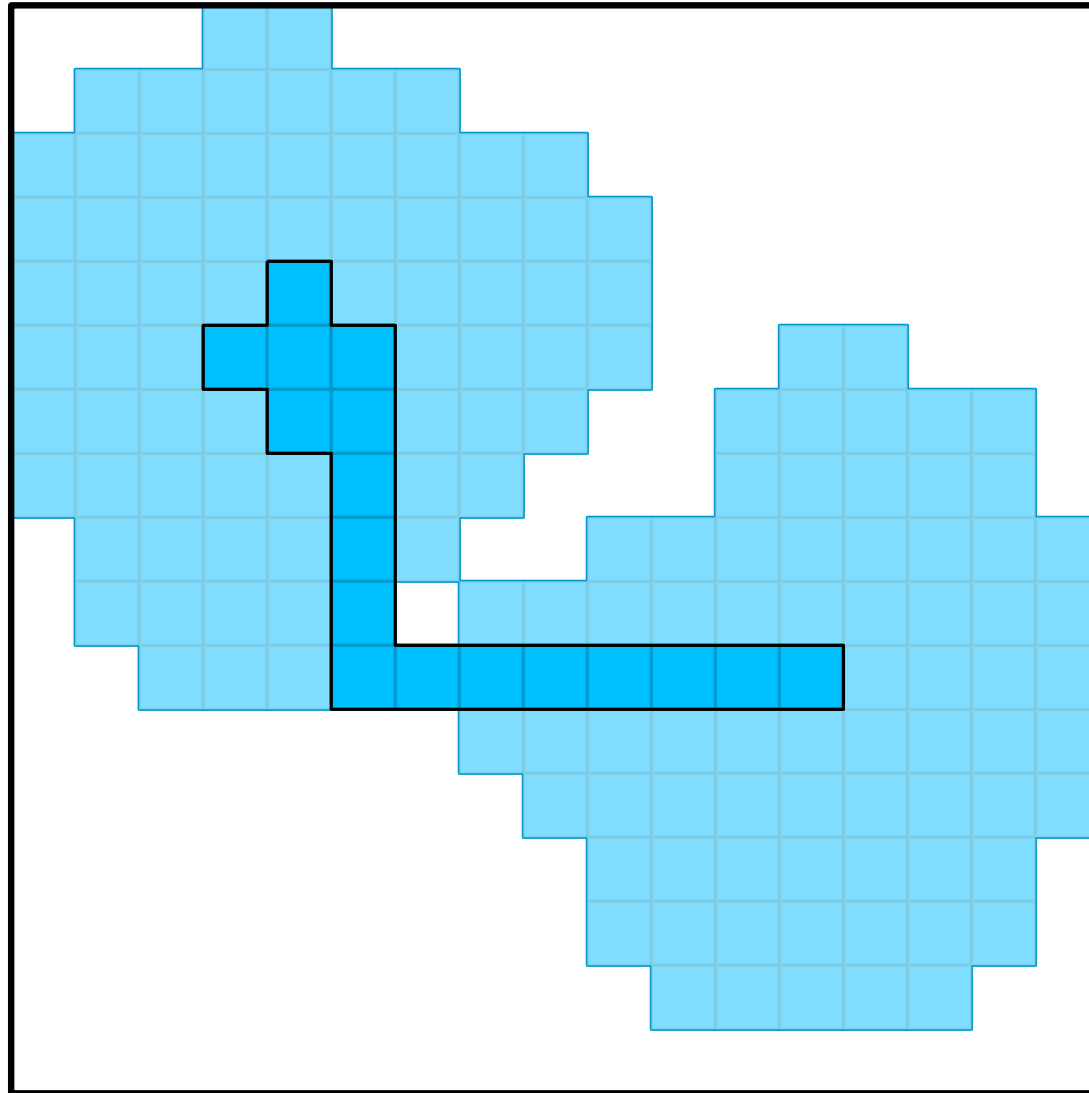
2 Colours (Free)



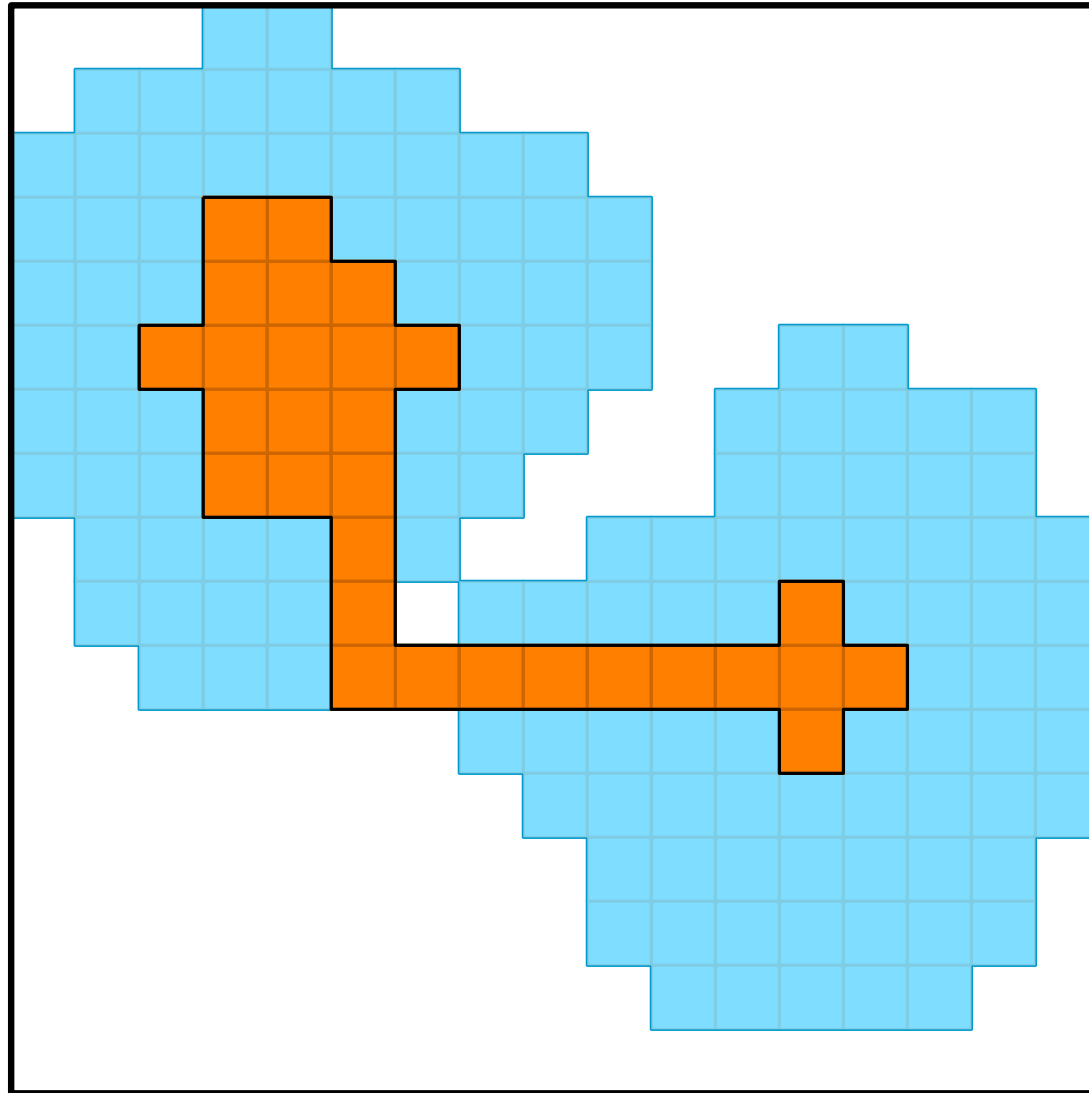
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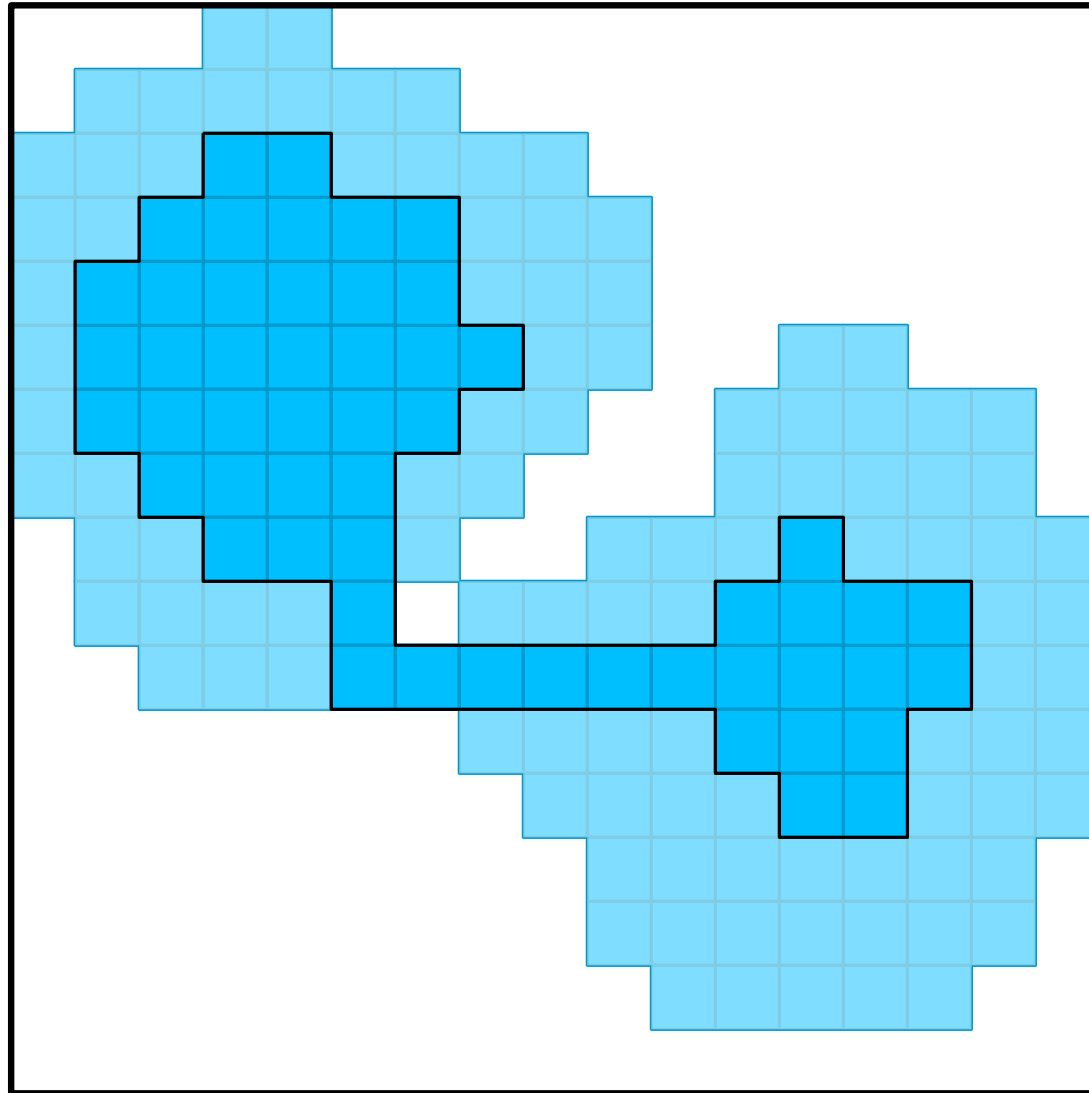
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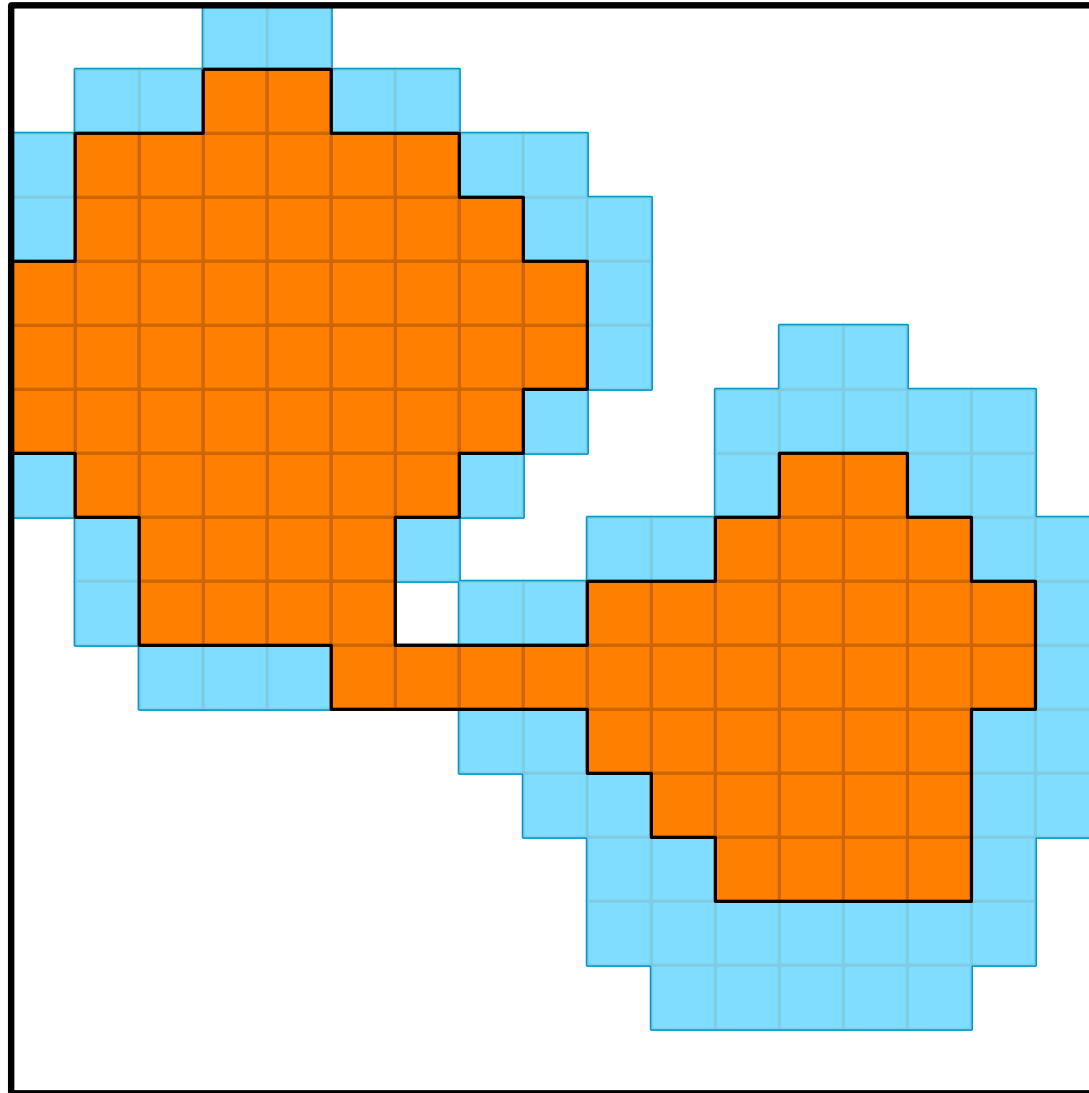
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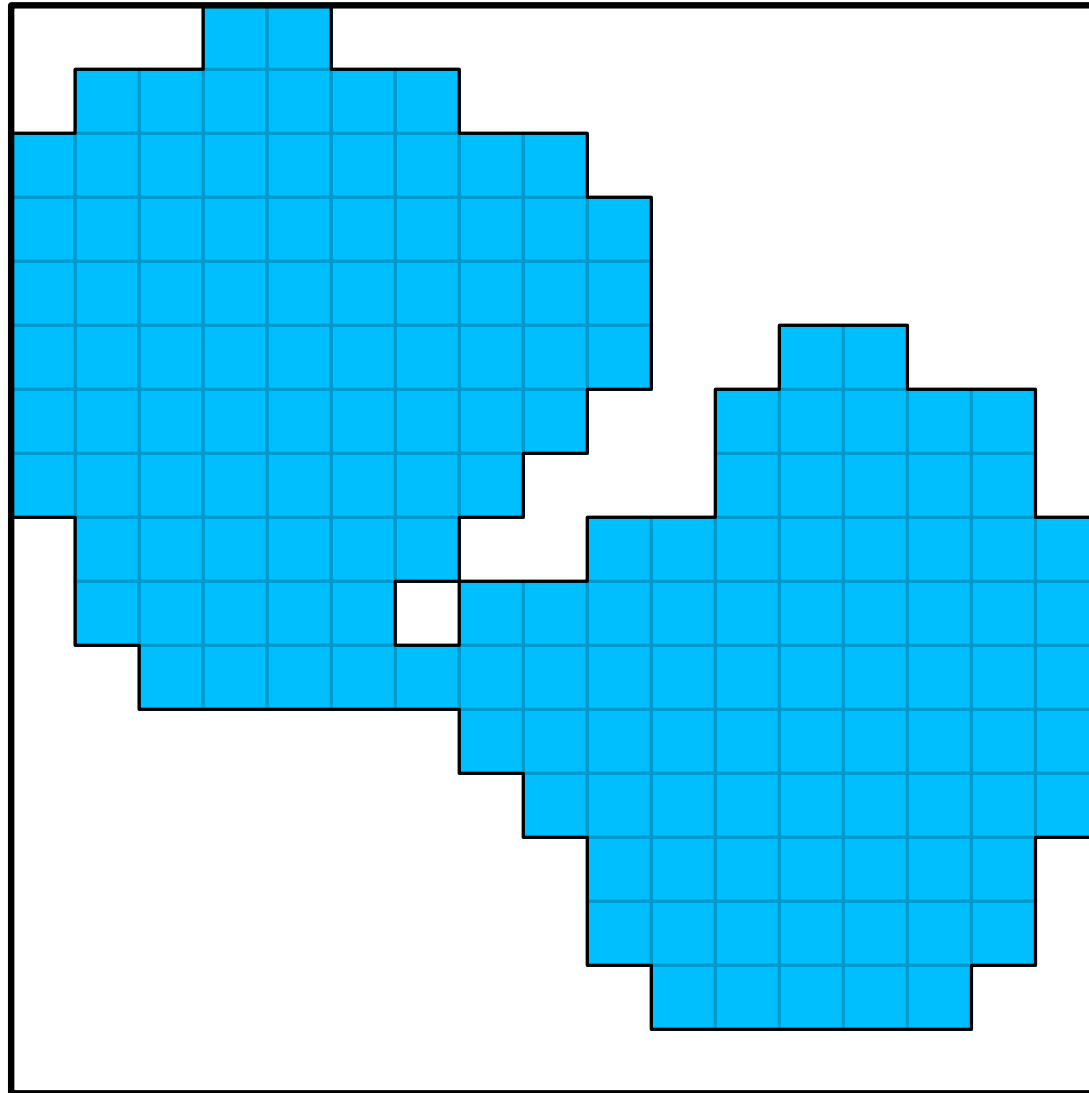
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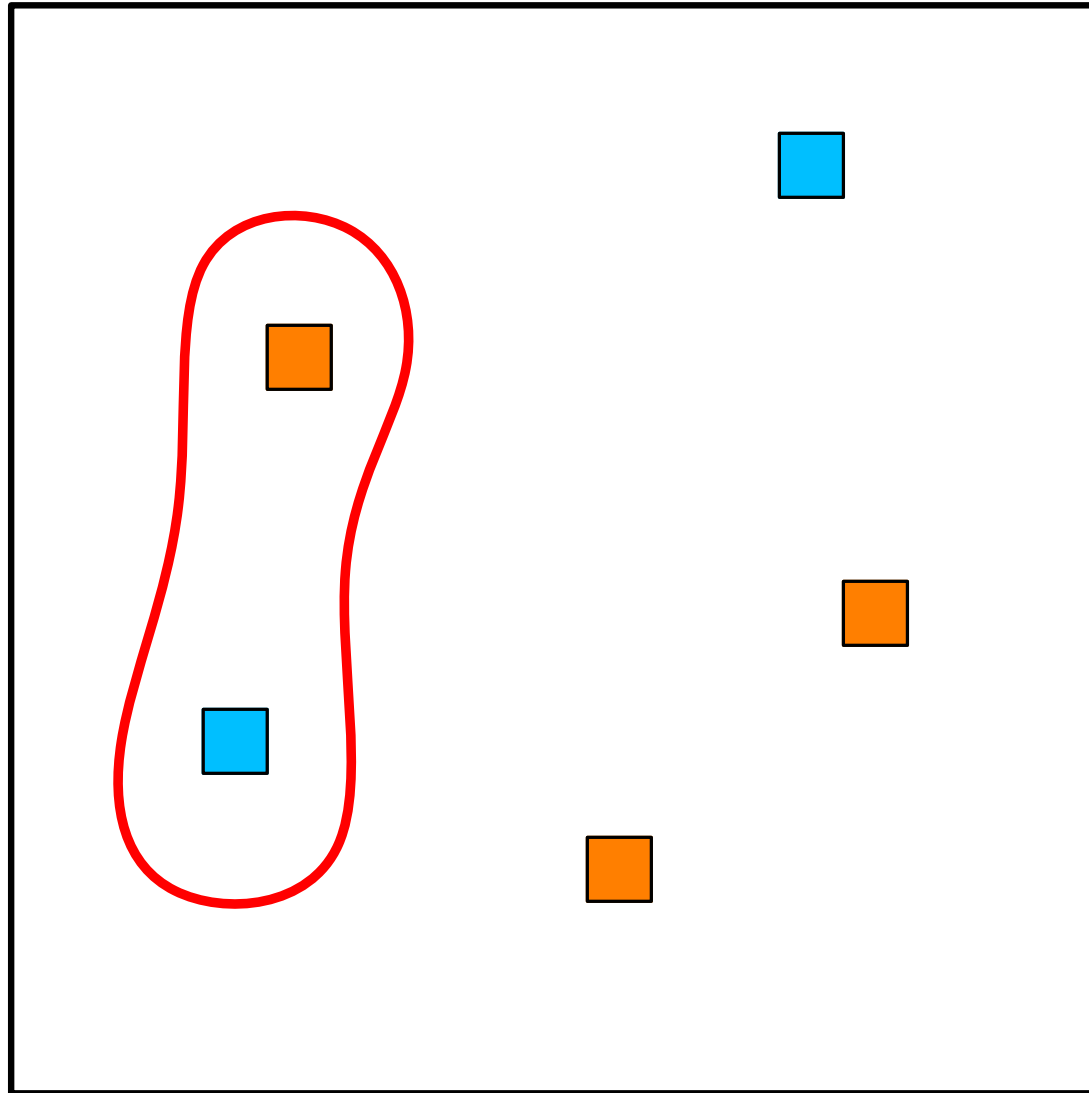
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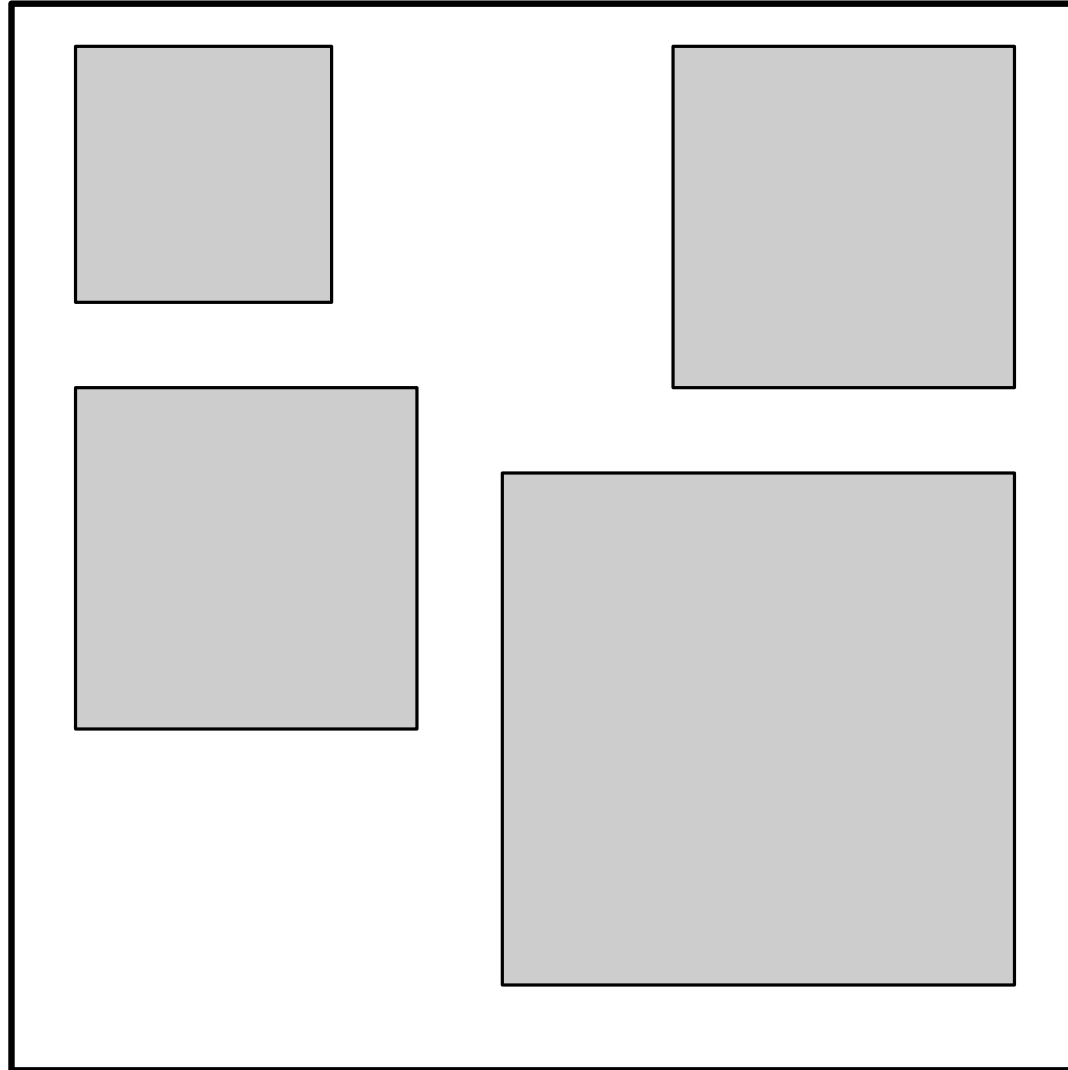
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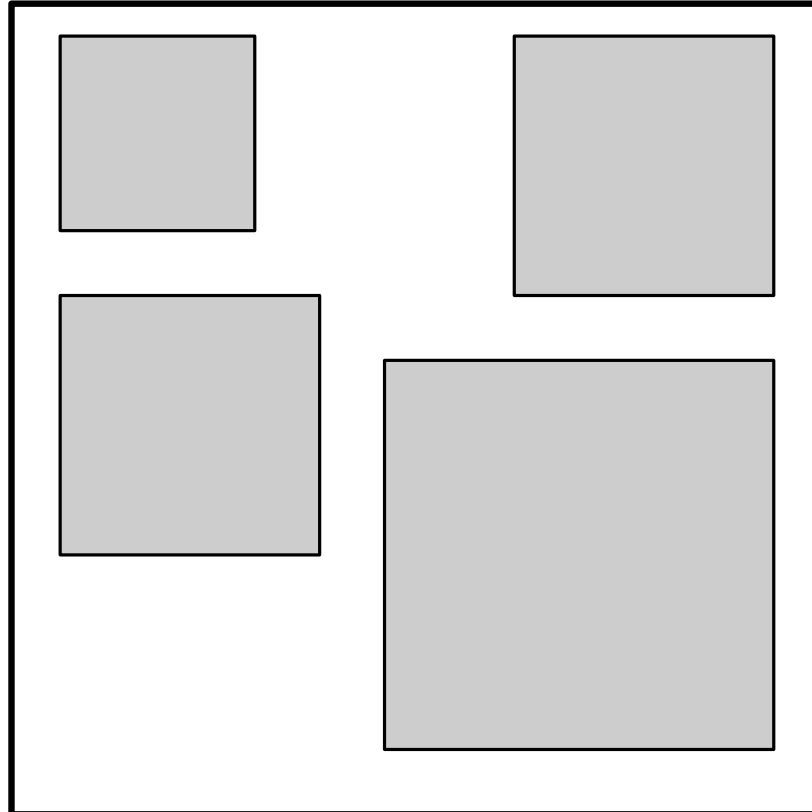
2 Colours (Free)



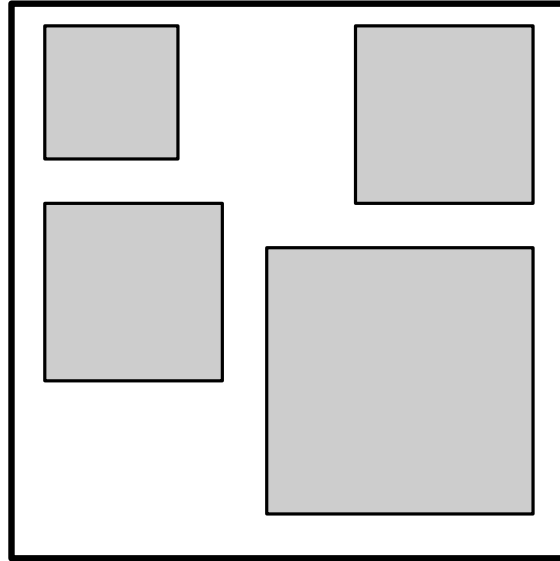
3 or More Colours (Free)



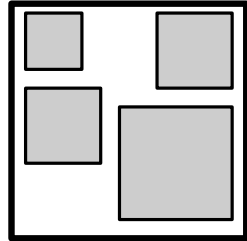
3 or More Colours (Free)



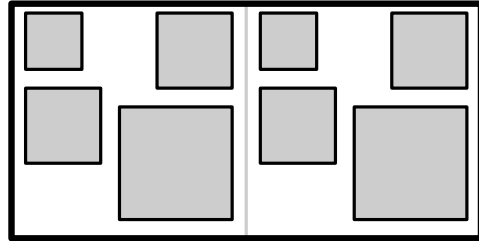
3 or More Colours (Free)



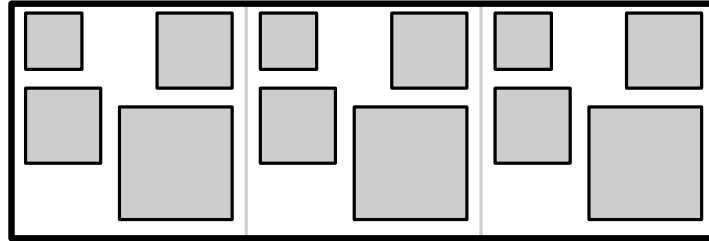
3 or More Colours (Free)



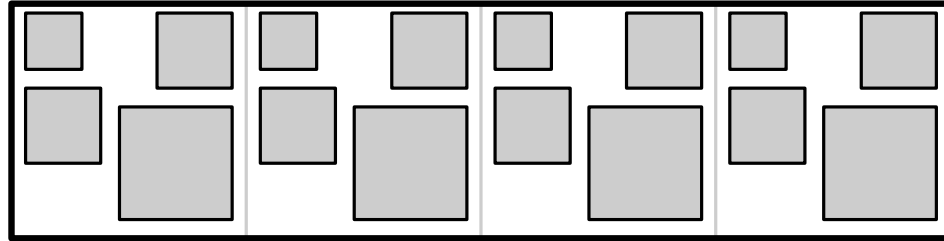
3 or More Colours (Free)



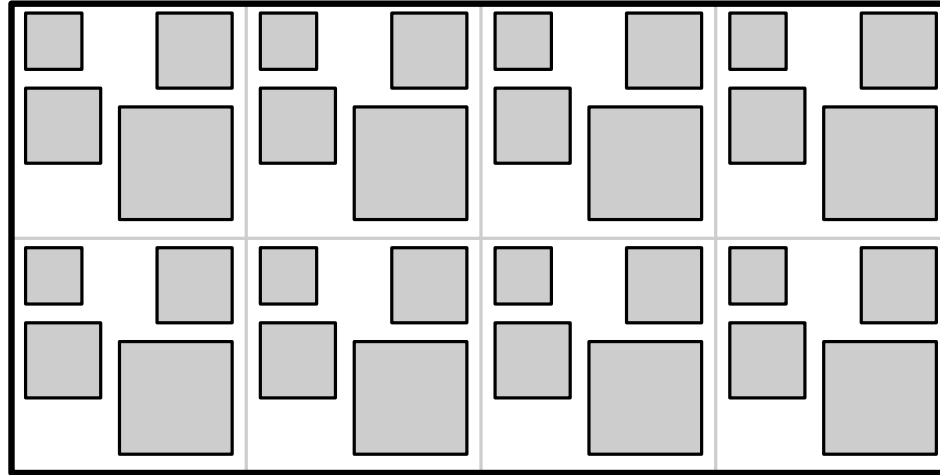
3 or More Colours (Free)



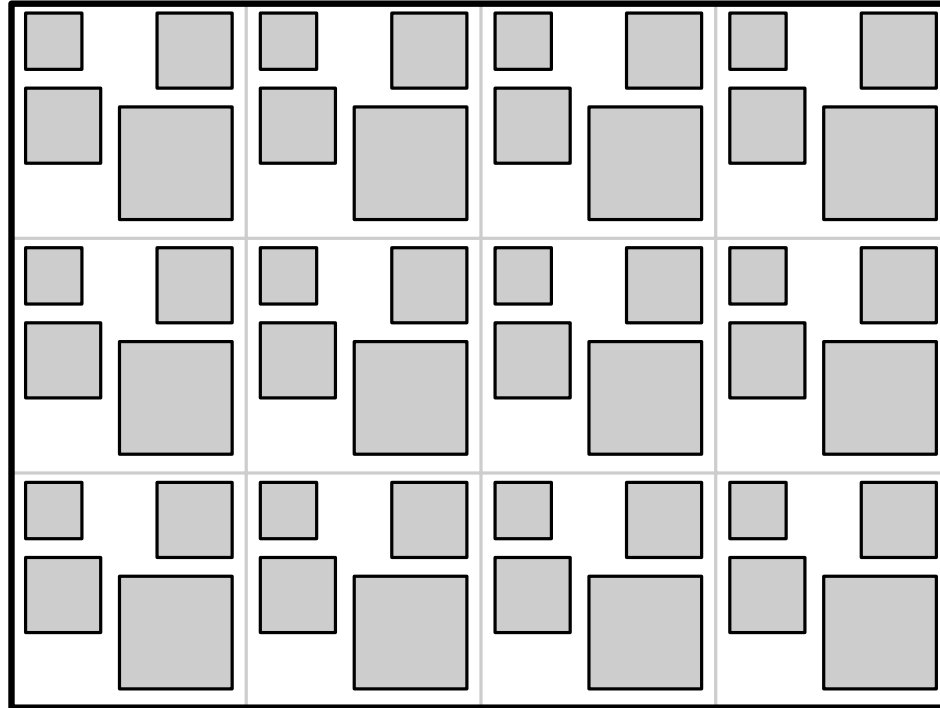
3 or More Colours (Free)



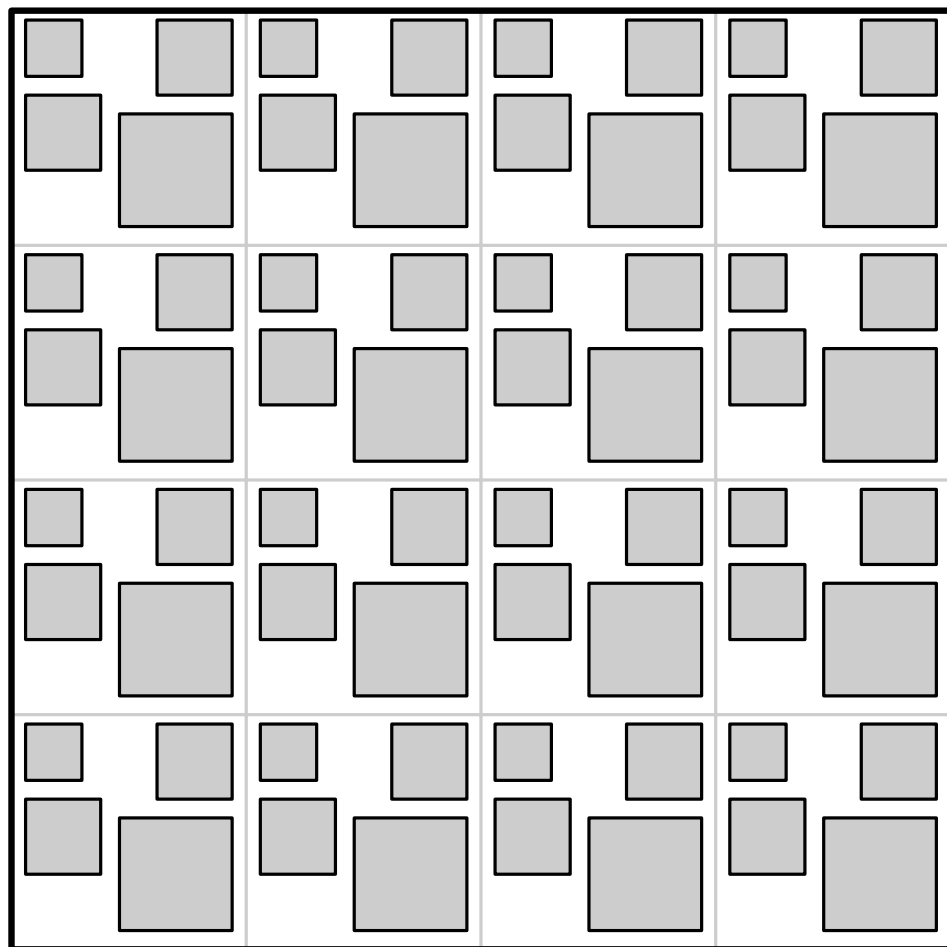
3 or More Colours (Free)



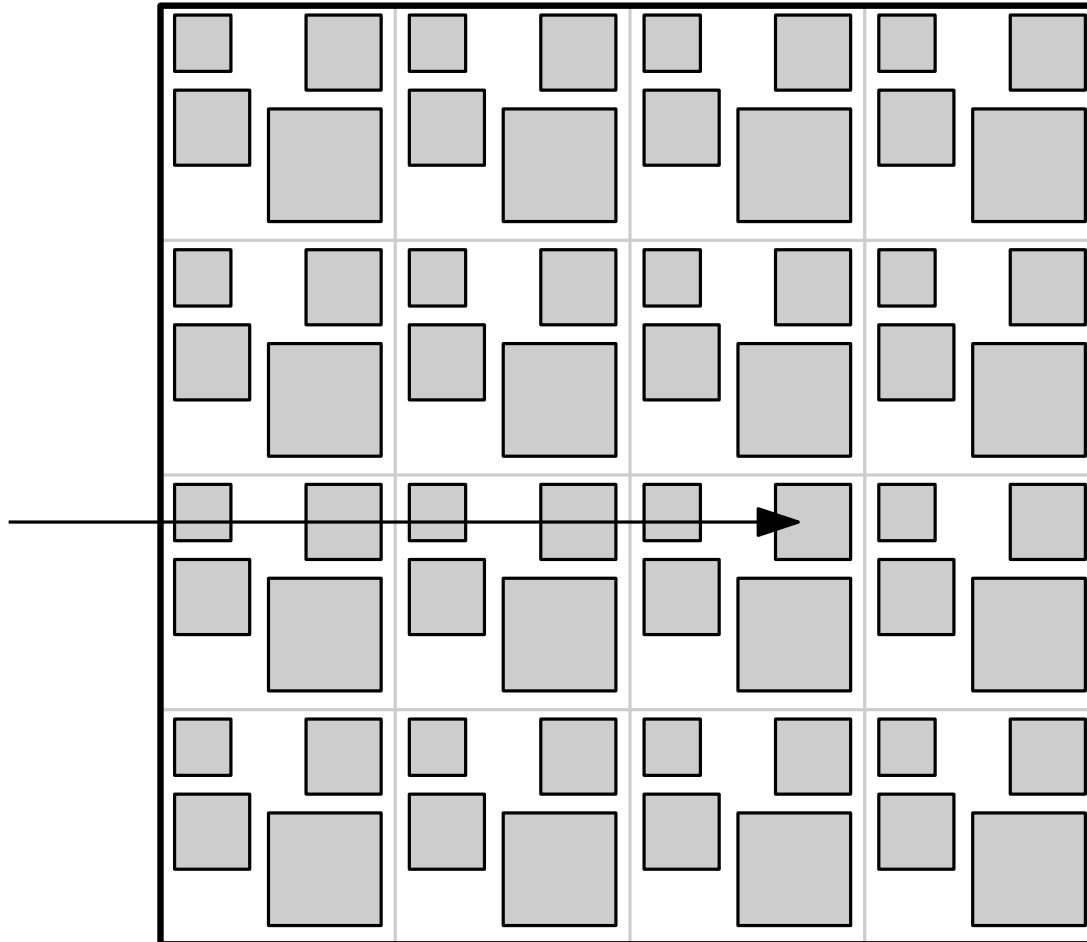
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